Chapter 4
Labor Demand

October 22, 2009

• Firms hire workers because consumers want to purchase a variety of goods and services; therefore, demand for workers is derived from the wants and desires of consumers, thus, called a derived demand.

• Central questions: how many workers are hired and what are they paid?
  - The demand for labor as:
    
    |                              | Product market is competitive | Product market is noncompetitive |
    |------------------------------|-------------------------------|---------------------------------|
    | Short run                   |                               |                                 |
    | Long run                    |                               |                                 |

  - Labor demand elasticity
  - The employment /hours trade-off

I. Static Theory of Labor Demand

Labor demand with one input

• Employers (firms) are perfect competitors both in the labor market and the product market, and maximize profits.

\[
\max_L \pi = P\phi(L) - WL,
\]

where \( P \) is the product price; \( \phi(L) \) is the production function, with \( \phi' > 0, \phi'' < 0 \); \( W \) is the nominal wage; \( L \) is the labor input.

\[F.O.C.: \quad P\phi'(L) - W = 0\]

\[
\phi'(L) - \frac{W}{P} = 0.
\]

\[
\phi'(L^*) = w, \quad (1)
\]

where \( w \) is the real wage, \( L^* \) is the profit-maximizing demand for labor.

• ___________-sloping demand curve

Totally differentiating in (1):

\[
\phi''(L^*)dL^* = dw \quad \Rightarrow \quad \frac{dL^*}{dw} = \frac{1}{\phi''(L^*)}.
\]
• If the product market is not perfect competition

\[ \text{Max}_L \pi = P(\phi(L))\phi(L) - WL \]

\[ F.O.C.: \ P(L)\phi'(L) + \frac{dP}{d\phi(L)} \frac{d\phi(L)}{dL} \phi(L) - W = 0 \]

\[ \phi'(L) \left[ 1 + \frac{1}{P} \frac{dP}{d\phi(L)} \phi(L) \right] - W = 0 \]

\[ \phi'(L) \left[ 1 + \frac{1}{d\phi(L)/P} \frac{dP}{\phi(L)} \right] = w \]

\[ \phi'(L^*) \left[ 1 - \frac{1}{\eta} \right] = w, \]

where \( \eta \geq 0 \) is the absolute value of the elasticity of product demand.

(i) As \( \eta \to \infty \), (2) reduces to (1).

(ii) Equation (2) shows that labor demand is more steeply sloped the less elastic is the demand for the product.

Labor demand with two inputs

• The production technology exhibits constant returns to scale, described by the linear homogeneous function \( F \), such that

\[ Y = F(K, L), \quad F_i > 0, \quad F_{ii} < 0, \quad F_{ij} > 0, \]

where \( Y \) is output, \( K \) is capital.

• Let the competitive product price to be $1 and the firm maximize profits:

\[ \text{Max}_{L,K} \pi = F(K, L) - wL - rK, \]

where \( r \) is the exogenous price of capital.

\[ F.O.C.: \ \frac{\partial F}{\partial L} = \frac{\partial F(K, L)}{\partial L} - w = F_L - w = 0 \quad \Rightarrow \quad F_L = w \]

\[ \frac{\partial F}{\partial K} = \frac{\partial F(K, L)}{\partial K} - r = F_K - r = 0 \quad \Rightarrow \quad F_K = r \]
\[ \frac{F_L}{F_K} = \frac{w}{r} \text{ or } \frac{F_L}{w} = \frac{F_K}{r} \]

- marginal rate of technical substitution (MRTS) = the factor price ratio
- The firm must adjust its labor and capital inputs so that the marginal cost of producing an added unit of output using \( L \) is equal to the marginal cost of producing an added unit of output using \( K \).
- The demand for \( L \) is a function of its own wage rate and the price of \( K \).

**The effect of a change in the price of input \( i \) on the demand for another input \( j \)**

- **If \( i \) and \( j \) are substitutes in production**
  
  As \( P_i \uparrow, \Rightarrow \ i \downarrow, \Rightarrow \) (a) \( j \uparrow \), substitution effect
  (b) \( j \downarrow \), scale effect

  If substitution effect dominates scale effect, \( i \) and \( j \) are gross substitutes.
  If scale effect dominates substitution effect, \( i \) and \( j \) are gross complements.

- **If \( i \) and \( j \) are complements in production**

  As \( P_i \uparrow, \Rightarrow \ i \downarrow, \Rightarrow \ j \downarrow \)

  Only scale effect, no substitution effect, \( i \) and \( j \) are gross complements.

- See Figure 4-10 and Figure 4-12.
II. Labor Demand Elasticity

• The own-wage elasticity of demand

\[ \eta_{ii} = \frac{\% \Delta E_i}{\% \Delta w_i} \]

The percentage change in the employment (E) for a category of labor \( i \) induced by a 1 percent increase in its wage rate (w).

• Marshall’s Rules of derived demand

   Labor demand is more elastic when:
   - Elasticity of substitution is greater
   - Elasticity of demand for the firm’s output is greater
   - The greater labor’s share in total costs of production
   - The greater the supply elasticity of other factors of production

• The cross-wage elasticity of demand

\[ \eta_{jk} = \frac{\% \Delta E_j}{\% \Delta w_k} \]

The percentage change in the demand for input \( j \) induced by a 1 percent increase in the price of input \( k \).

\[ \eta_{jk} > 0, \quad j \text{ and } k \text{ are gross } \quad \text{.} \]
\[ \eta_{jk} < 0, \quad j \text{ and } k \text{ are gross } \quad \text{.} \]
• The elasticity of substitution
  - The effect of a change in relative factor prices on relative inputs of two factors, holding output constant. Intuitively, the elasticity measures the ease of substituting one input for the other without changing output, i.e., to measure the curvature of the isoquant.
  - The elasticity of substitution between capital and labor:

\[
\sigma = \frac{\% \Delta \left( \frac{K}{E} \right)}{\% \Delta \left( \frac{w}{r} \right)}
\]

?Add more: Hamermesh., p-24 (?)

• Estimates of these elasticity: general findings (E&S, p-101, p-107)

• Policy applications
  Minimum wage (B, p-136)
  Technological change (E&S, p-116)

III. The Employment / Hours Trade-off, and the Cost of Labor

• So far, the term of “labor” has not been strictly defined. Actually, a firm can produce a given level of output with various combinations of the number of employees hired and the number of hours worked per week. Presumably, more workers, fewer hours; less workers, more working hours.

• The distinction between workers and hours is crucial in evaluating the impact of some employment policies. For example:
  - An increase in employer-provided health insurance premium would discourage the firm from adding to its workforce.
  - Legislation mandating employers to pay an overtime premium mainly affects the cost of lengthening the workweek.

How does a firm determine its optimal employment / hours combination?

• Essentially, we have implicitly assumed for each type of labor \( i \) that
\[ L_i = E_i H_i, \]
where \( E \) is employment, and \( H \) is the hours working per time period.

- The labor costs structure
  The main distinction is between costs that vary with \( E \) (fixed costs, measured on a per-worker basis as \( F \)) and those that vary with hours (variable costs \( V \)), as shown in Figure A.

**Figure A**

- The typical firm faces fixed costs per period:
  \[ EF = E (R + [r + q T]). \]
  
  \( r \): the borrowing rate, exogenous to the firms’ choice between workers and hours.
  
  \( q \): the quit rate, exogenous (maybe endogenous) to the firms’ choice between workers and hours.
  
  \( r + q \): per-period of costs during the tenure of the worker in the plant (one–time costs).
  
  Higher values of \( R, r, q, \) or \( T \) can result in higher value of \( F \).

- If workers have identical tastes, the firm will face a wage:
  \[ w = w(H), \quad w' > 0. \]
• The firm’s labor costs are: Labor cost = $EHw(H) + EF$.

• To make the analysis easier, let’s assume the choices about workers and hours are separable from capital (Noted, this assumption is not always correct).

• What is the shape of the labor aggregator, $L = L(E, H)$?

Let $C^0$ be the labor cost constraint that the firm is subject to maximize its output, and assume that $\frac{\partial H}{\partial C^0} = 0$, that is, the firm’s optimal hours is independent of scale. This assumption requires that $L = \phi_1(E)\phi_2(H)$, $\phi_i' > 0$. For example, $L = aE^b g(H)$; $a$, $b > 0$, and $g' > 0$.

• $\max_{L,H} \mathcal{J} = L(E, H) + \lambda[C_0 - EHw(H) - EF]$

F.O.C.: $\frac{\partial \mathcal{J}}{\partial E} = LE - \lambda[Hw + F] = 0$

$\frac{\partial \mathcal{J}}{\partial H} = LH - \lambda[Ew + EHw'] = 0$

$$\Rightarrow \frac{L_E}{L_H} = \frac{Hw + F}{Ew + EHw} = \frac{Hw + F}{Ew \left[1 + w' \frac{H}{w}\right]} = \frac{Hw + F}{Ew[1 + \varepsilon]}.$$ (3)

where $\varepsilon$ is the elasticity of wages with respect to hours.

• From (3), the demand functions of $E$ and $H$ are

$E^* = E(F, \varepsilon, C^0),$

$H^* = H(F, \varepsilon, C^0),$

where the superior sings denote the effects on employment or hours of increasing the parameter in question.