



## Research Note

## Forecasting technology diffusion with the Richards model

Yorgos D. Marinakis\*

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## ABSTRACT

The Richards model has a shape parameter  $m$  that allows it to fit any sigmoidal curve. This article demonstrates the ability of a modified Richards model to fit a variety of technology diffusion curvilinear data that would otherwise be fit by Bass, Gompertz, Logistic, and other models. The performance of the Richards model in forecasting was examined by analyzing fragments of data computed from the model itself, where the fragments simulated either an entire diffusion curve but with sparse data points, or only the initial trajectory of a diffusion curve but with dense data points. It was determined that accurate parameter estimates could be obtained when the data was sparse but traced out the curve at least up to the third inflection point (concave down), and when the data was dense and traced out the curve up to the first inflection point (concave up). Rogers' Innovation I, II and III are discussed in the context of the Richards model. Since  $m$  is scale independent, the model allows for a typology of diffusion curves and may provide an alternative to Rogers' typology.

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## 1. Introduction

Technology diffusion is widely accepted as tracing a sigmoidal trajectory that resembles a biotic growth curve [1,2]. Accordingly, empirical growth models are commonly fit to technology diffusion data. Much of the activity surrounding empirical growth models relates to model selection, where for example the Fisher–Pry model [3] is appropriate to technology diffusion in general and the Gompertz [4] as a mortality model is appropriate to cases of technology diffusion involving replacement [5].

The Richards model [6] is an empirical model developed for fitting growth data. Through the use of a shape parameter  $m$  that enables the curve to stretch or shrink, the Richards model encompasses the Gompertz, Fisher–Pry and every other imaginable sigmoidal model (Fig. 1). When  $m = 0$ , the model approximates the exponential growth function. When  $m = 0.67$ , the model behaves like the von Bertalanffy [7]. When  $m$  approaches 1, the model behaves like the Gompertz. When  $m = 2$ , the model behaves like the Logistic model [8]. Thus one of the advantages of the Richards model is that it, in effect, selects the model for you.

The Richards model has been investigated in at least two white papers as a tool for technology forecasting [9,10], but it has not been reported in the peer-reviewed literature in the context of diffusion of technology diffusion or diffusion forecasting. Neither has its behavior under a variety of data qualities been examined. This article demonstrates the ability of a modified Richards model to fit a variety of technology diffusion curves that would otherwise be fit by the Bass [11], Gompertz, Logistic, and other models.

\* Tel.: +1 505 867 6115.

E-mail address: George@journalofsustainability.com.

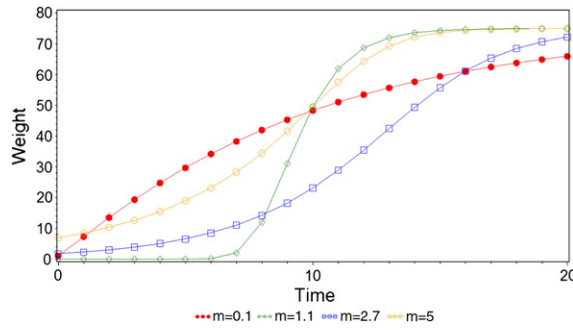


Fig. 1. The Richards model at various m-values.  $W^\infty$ ,  $T^*$ , and k also vary between curves.

2. Methods

The Richards model was introduced in 1959 in the context of plant growth [6]. Richards conceived of the model as an extension to the von Bertalanffy model, but he recognized its ability to emulate the Gompertz and Logistic as well. The model has been modified and reparameterized by several researchers. As modified by Sugden et al. [12], the model is:

$$W_t = W^\infty \left[ 1 - (1-m) \exp \left[ -k(t-T^*) / m^{m/(1-m)} \right] \right]^{1/(1-m)}$$

where  $W_t$  is the weight or growth at time  $t$ ,  $W^\infty$  is the asymptotic weight,  $k$  is the maximum relative growth rate per unit time,  $T^*$  is the maximum rate of growth per unit time, and  $m$  is a shape parameter with the property that  $m^{1/(1-m)}$  is the relative weight at time  $T^*$ .

In terms of technology diffusion,  $W^\infty$  is the asymptotic weight, i.e., the asymptotic or maximum level of diffusion achieved.  $k$  is the maximum relative growth rate per unit time; when applied to diffusion data,  $k$  signifies the maximum diffusion per unit time relative to the size of the population. As shown in Fig. 2a, the  $k$ -value is associated with the lagging effect: the higher the  $k$ -value, the longer the lag.  $T^*$  is the maximum rate of diffusion per unit time. The  $m$ -value is the shape parameter that determines the position of the concave up–concave down inflection point. As shown in Fig. 2b, all other parameters held equal, the relation

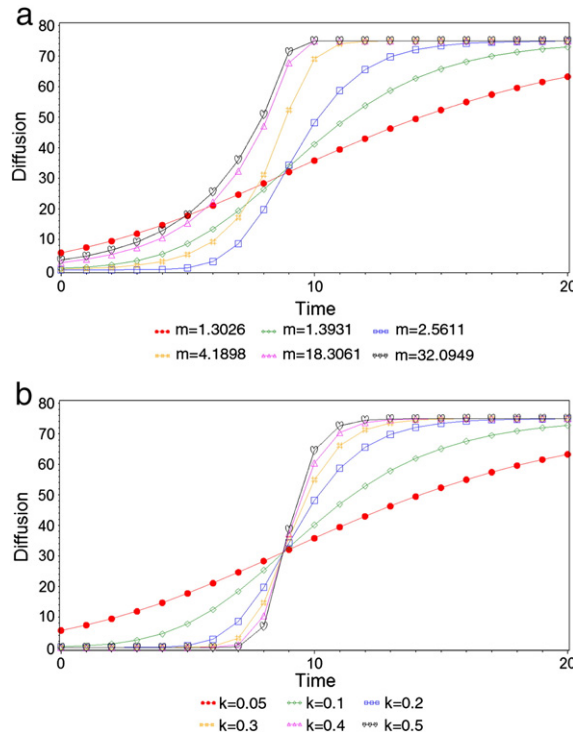


Fig. 2. a. Shapes of curves in Table 1, all other parameters held equal. b. Richards model curves for  $m = 1.3026$ , but  $k$  varies from 0.05 to 0.1.

between an initial lag and the  $m$ -value is complex. As the  $m$ -value increases, the curve becomes more sigmoidal, moving from something close to a straight line at  $m = 1.3026$  through a series of increasingly steep sigmoidal shapes.

The Richards model has been used in contexts other than body growth. White and Marinakis [13] for example used a reparameterized Richards model [14] to quantify nitrogen mineralization patterns. That particular form of the equation was not used here because it is restricted to phase space, i.e., it fits  $dW/dt = f(W)$ , such that it cannot be used to forecast in the temporal domain.

The model was fit to the data with Proc Nlin in SAS PC 9.1.3. Proc Nlin requires a grid search, and the failure of a model to converge often indicates the grid search was too restricted in some sense. Note that it is necessary to renormalize the start time of the data to zero.

The model was run on several datasets previously analyzed in the peer-reviewed literature (Table 1). P-values and graphical comparisons of the computed models and the data are presented to visually demonstrate the accuracy of the model (Fig. 3a–f).

### 3. Results

#### 3.1. Fitting technology diffusion data with the Richards model

The Richards model accurately modeled all of the datasets (Table 1, Fig. 3a–f). In all cases, regarding the model, the probability of the F ratio being as large or greater (under the null hypothesis) than the value that was obtained was  $P = 0.0001$ . Root mean squared errors (MSE) were always smaller than the standard deviation of the data.

The Richards model fit the following data sets:

Rogers [1] page 45 on the diffusion of diffusion research publications (Fig. 3a, Table 1) with  $m = 1.30$ ,  $W^\infty = 3996.7$ ,  $T^* = 28.47$ , and  $k = 0.0395$ ;

Rogers [1] page 258 on the adopters of hybrid seed corn in two Iowa communities (Fig. 3b, Table 1) with  $m = 4.19$ ,  $W^\infty = 257.5$ ,  $T^* = 11.22$ , and  $k = 0.2170$ ;

Chu et al. [15] and Wu and Chu [16] on the diffusion of mobile telephony in Taiwan (Fig. 3c, Table 1) with  $m = 2.56$ ,  $W^\infty = 104.5$ ,  $T^* = 5.21$ ,  $k = 0.3306$ , and  $P = 0.0001$ . The authors had fit the data using Bass, Gompertz, Logistic and ARMA models;

The chemical patent data set of Andersen [17] (Fig. 3d, Table 1) with  $m = 32.10$ ,  $W^\infty = 3522.9$ ,  $T^* = 18.77$ , and  $k = 0.0366$ ;

The CATV data set from Porter et al. [5] with  $m = 18.31$ ,  $W^\infty = 48.36$ ,  $T^* = 30.29$ , and  $k = 0.1045$  (Fig. 3e, Table 1). Those authors had used the data to demonstrate the Fisher–Pry and Gompertz models;

Michalakelis et al. [18] on mobile telephony in Greece with  $m = 1.39$ ,  $W^\infty = 12,703,310$ ,  $T^* = 6.8$ , and  $k = 0.1608$  (Fig. 3f, Table 1). In that article the authors fit the data using Bass, Gompertz, Logistic, Box-Cox [19], FLOG [20] and TONIC [21] models.

To further test the validity of the Richards model, the model was fit to the first half of each data set. The results support a finding of validity (Table 1). In four out of six cases, RMSE is less than the standard deviation of the data, and  $Pr < F$  at less than 0.05.

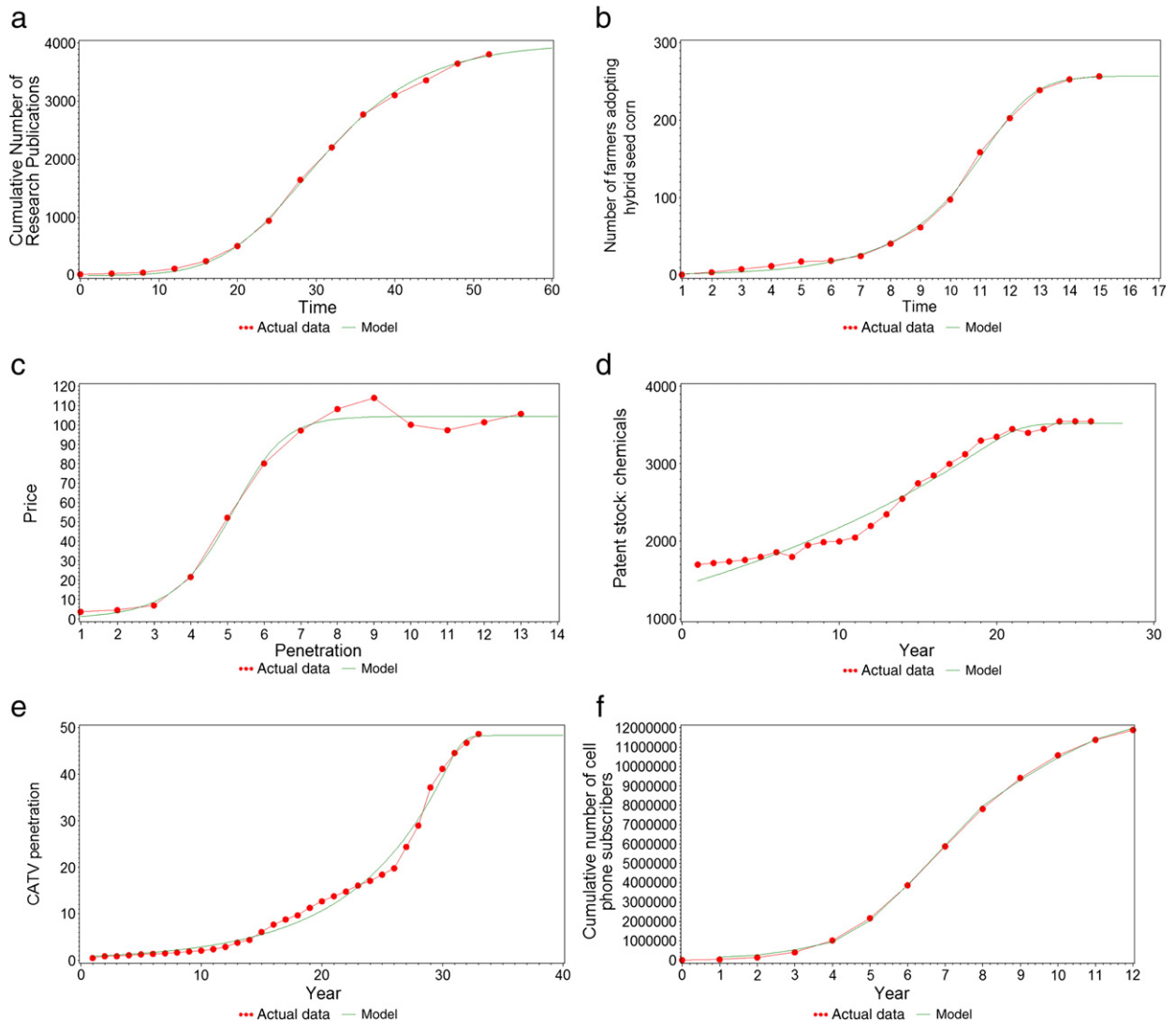
#### 3.2. Forecasting

As discussed in Ref. [10], logistic and Gompertz models are the most commonly used to fit diffusion data. Compared to the Richards model, these models are relatively inflexible. They force a concave up–concave down inflection point at 50% and 37% of

**Table 1**

Richards model parameters.  $Pr > F$  refers to the probability of the F ratio being as large or greater (under the null hypothesis) than the value that was obtained. Two results are shown for each data set. The first result corresponds to the analysis of the entire data set. The second result is a validity analysis that corresponds to the analysis of the first half of the data set.

	$m$	$W^\infty$	$T^*$	$k$	Converged?	$Pr > F$	RMSE	Data standard deviation	RMSE < StdDev?
Rogers [1], page 45 (diffusion research publications), days 1–52;	1.30	3996.7	28.47	0.0385	Yes	0.0001	45.90	2243497.05	Yes
Using only days 1–28.	0.1499	17132.1	60.1	0.00124	No	0.7368	810.53	579.28	No
Rogers [1], page 258 (adopters of hybrid seed corn in two Iowa communities), days 1–151	4.19	257.5	11.22	0.2170	Yes	0.0001	4.09	99.95	Yes
Using only days 1–7.	2.6087	483.4	16.5	0.0947	No	0.0077	3.30	8.70	Yes
Chu et al. [15], (mobile telephony), days 1–131	2.56	104.5	5.21	0.3306	Yes	0.0001	4.99	44.26	Yes
Using only days 1–7.	2.23	103.9	5.1	0.3207	Yes	0.0001	2.27	38.79	Yes
Andersen [17], Fig. 2, chemical patents, days 1–261	32.10	3522.9	18.77	0.0366	Yes	0.0001	115.04	717.77	Yes
Using only days 1–13.	23.9	3341.7	40.0	0.0109	No	0.0001	124.38	198.13	Yes
Porter et al. [5], page 142, 175 ff., CATV, days 1–331	18.31	48.36	30.29	0.1045	Yes	0.0001	1.41	14.91	Yes
Using only days 1–33.	5.0	50.0	30.0	0.1100	No	0.0348	2.58	2.02	No
Michalakelis et al. [18], days 1–12;	1.39	12,703,310	6.80	0.1608	Yes	0.0001	1.23E5	4.65E6	Yes
Using only days 1–6.	6.2	14,757,550	10	0.2100	No	0.2018	1.04E6	1.4E6	Yes



**Fig. 3.** a. Data from Rogers page 45 [1]. b. Data from Rogers page 258 [1]. c. Data from Chu et al. [15]. d. Data from Andersen [17]. e. Data from Porter [5]. f. Data from Michalakelis et al. [18].

the asymptote, respectively. Moreover, the logistic is symmetric about that inflection point, and the Gompertz always has a larger speed of adoption after that inflection point. The Richards model has no fixed concave up–concave down inflection point, and it allows for any and all types of asymmetry about that inflection point.

Sokele [9] notes that the Richards model is not suitable for modeling market adoption immediately after a product is introduced because as  $t \rightarrow -\infty$ , the Richards model goes to zero; and that the Richards model is more flexible than the logistic or Bass in fitting data with an asymmetric inflection point. Data requirements for achieving accurate parameter estimates will now be examined.

The performance of the Richards model in forecasting was examined by analyzing fragments of data computed from the model itself, where the fragments simulated either an entire diffusion curve but with sparse data points, or only the initial trajectory of a diffusion curve but with dense data points (Table 2; figures not shown). It was determined that accurate parameter estimates could be obtained when the data was sparse but traced out the curve at least up to the third inflection point (concave down; the midpoint is the second inflection point), and when the data was dense and traced out the curve up to the first inflection point (concave up).

For example, with data generated by the model using the same parameters as those from Rogers [1] page 45 (Fig. 3a, Table 1;  $m = 1.3026$ ,  $W^\infty = 3996.7$ ,  $T^* = 28.4696$ , and  $k = 0.0395$ ), with only iterations that traced out the data up to approximately the midpoint ( $w = 3165$ ;  $i = 1$  to 40 by 1), the model failed to converge and the parameter estimates were marginally accurate ( $m = 4.7703$ ,  $W^\infty = 3632.2$ ,  $T^* = 78.8723$ ,  $k = 0.0121$ ). But using 170 iterations that traced out the data to just below the first inflection point ( $i = 1$  to 17 by 0.1), the model converged and the parameter estimates were perfectly accurate. With lesser amounts of data (e.g.,  $i = 1$  to 16 by 0.1), the model failed to converge or provide accurate parameter estimates.

**Table 2**

Experimental design for forecasting. The bold entries correspond to an analysis of the entire data set. The following entries correspond to analyses of subsets and transformations of that data.

Data generated using parameters from:	m	$W^\infty$	$T^*$	k	Converged?
<b>Rogers [1], page 45) (diffusion research publications):</b>	<b>1.30</b>	<b>3996.7</b>	<b>28.47</b>	<b>0.0385</b>	<b>Yes</b>
Data up to approximately the midpoint;	4.7703	3632.2	78.8723	0.0121	No
Data to just below the first inflection point, dense data;	1.30	3996.7	28.47	0.0385	Yes
<b>Rogers [1], page 258 (adopters of hybrid seed corn in two Iowa communities):</b>	<b>4.19</b>	<b>257.5</b>	<b>11.22</b>	<b>0.2170</b>	<b>Yes</b>
Data up to and including the first inflection point;	4.1898	257.5	11.2243	0.0395	Yes
Using data up to and including the first inflection point but decreasing the data density;	87.3543	107.7	6.6743	0.0307	No
Using only the first five data points (do $i = 1$ to 5 by 1).	5.1975	226.1	25.5092	0.1225	No

In another experiment, using relatively dense data generated by the model using the same parameters as those from Rogers [1] page 258 (Fig. 3b, Table 1;  $m = 4.1898$ ,  $W^\infty = 257.5$ ,  $T^* = 11.2243$ , and  $k = 0.2170$ ), statistically significantly fits and nearly perfectly accurate parameters ( $m = 4.1898$ ,  $W^\infty = 257.5$ ,  $T^* = 11.2243$ ,  $k = 0.0395$ , and  $P = 0.001$ , converged) were obtained using the data that traced a curve up to and including the first inflection point (eight data points; i.e., do  $i = 1$  to 8 by 1). Using data up to and including the first inflection point but decreasing the density (i.e., do  $i = 1$  to 8 by 2) resulted both in inaccurate parameter estimates ( $m = 87.3543$ ,  $W^\infty = 107.7$ ,  $T^* = 6.6743$ , and  $k = 0.0307$ ) and a failure of the regression to converge. Reasonably accurate but statistically insignificant parameter estimates were obtained using data that traced out even less of the curve. For example, using only the first five data points (do  $i = 1$  to 5 by 1), the regression produced  $m = 5.1975$ ,  $W^\infty = 226.1$ ,  $T^* = 25.5092$ ,  $k = 0.0145$ , and  $P = 0.1225$ , though the model failed to converge. Increasing the density of the data (do  $i = 1$  to 5 by 0.1) did not improve the performance. Again it can be seen that accurate parameter estimates can be obtained from relatively dense data that traces out the curve up to at least the first inflection point.

For the data from the Taiwan telecommunications industry [15,16] (Fig. 3c, Table 1), the model was run again using only the first four data points, which points traced the curve up to the first inflection point. The model converged but the parameters were inaccurate. The model did not both converge and provide accurate parameters until the addition of the seventh data point, which traced the curve up to the third inflection point. However, using the data computed from the model using the above referenced values for the parameters, with only the first four data points sparsely populated as in the original data set ( $i = 1$  to 4 by 1), the model converged with perfectly accurate parameters. Four data points up to the first inflection point were sufficient here, but they were insufficient in other data sets, e.g. Rogers [1] page 45. Thus the difference between relatively dense data and relatively sparse data is a difficult line to draw. One must look for convergence to determine whether parameters are accurate or not. In addition, as a safety check the analyst is advised to use the derived parameters ( $m$ ,  $W^\infty$ ,  $T^*$ , and  $k$ ) to compute data to visually compare against the original data.

#### 4. Discussion

Rogers [1] identifies three types of technology curves, which he refers to as Innovation I for rapid adoption, Innovation III for slower adoption, and Innovation II between them (Fig. 4). When placed adjacently the distinctions between these curves can be discerned, but the theory does not tell us how to categorize any given curve in isolation. We can begin to approximate these types with Richards model by using decreasing  $m$ -values, all other parameters held equal (Fig. 5), in which case the lowest  $m$ -value has the slowest take off (i.e., the lowest slope in the initial portion of the curve). The shapes of the curves analyzed in this article (Table 1) can be similarly depicted (Fig. 6), all other parameters held equal. Such an analysis however reveals that simply changing the  $m$ -value fails to capture other features of the transition between Innovation I, II and III, namely a much steeper take off (slope) in Innovation I. To capture that feature, it is necessary to alter other parameters along with the shape parameter  $m$ . As shown in Fig. 2a, in which the  $k$ -value is changed but all other parameters are held equal, the higher the  $k$ -value, the steeper the take off. In

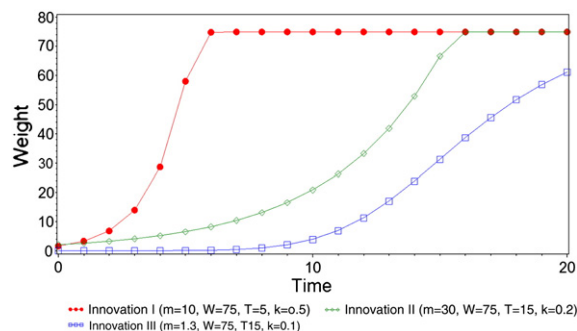


Fig. 4. Types of innovation per Rogers [1].

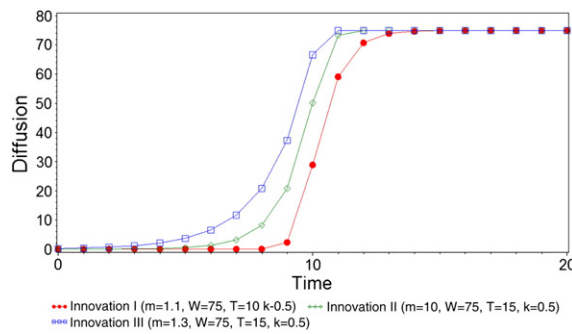


Fig. 5. Approximating Rogers' [1] innovation types with the Richards model by using decreasing  $m$ -values, all other parameters held equal.

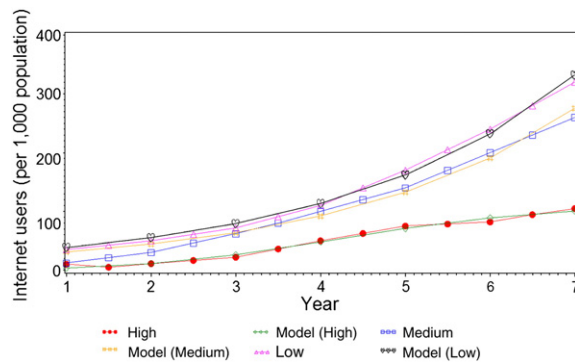


Fig. 6. Average diffusion of internet use, by income inequalities. Data from Noh and Yoo [22].

Fig. 4, by varying  $m$ ,  $T^*$  and  $k$ , we have a closer facsimile of Rogers' three types of Innovation, vis-à-vis Fig. 5 in which only  $m$  is varied. Thus the differences between the rapid adoption of Innovation I and the slower adoption of Innovations II and III are a complex of shapes ( $m$ ) that vary unsystematically combined with decreasing growth rates ( $T^*$  and  $k$ ). It may be that Rogers' Innovations I, II and III are oversimplifications. Or it may be that the Richards model (at least the four parameter version) is not suited to generalizing Rogers' types.

If Rogers' typology is set aside for a moment, it can be seen that the  $m$ -values provide a scale-independent method for characterizing technology diffusion curves, such that the  $(m, W^\infty, T^*, k)$  phase space (or subsets thereof) may provide the basis for an alternative to Rogers' typology. The data presented herein represent a wide variety of diffusion processes, with  $m$ -values varying from 1.3 to 32. The  $m$  value ( $m = 1.39$ ) for the data in Michalakelis [18] was close to that of Rogers [1] page 45 ( $m = 1.30$ ). Despite a difference of four orders of magnitude in the asymptotic growth parameter values, and differences by a factor of four in their growth rates, they trace out a similar shape ( $m \sim 1.3$ ) in their respective reference frames. This similarity may indicate similar processes. Thus one potentially informative analysis would be to examine  $m$  vs.  $\ln(T^*)$  for a large number of cases.

The  $m$ -value facilitates both hypothesis testing and quantitative comparisons. For example, Noh and Yoo [22] compare the average diffusion of internet use by income inequalities (Fig. 6). They show that “countries with higher income inequality lagged behind those with lower income inequality in internet diffusion.” An analysis with the Richards model enables us to see that the three curves have three different  $m$ -values, clustered into two sets (Table 3, Fig. 6). A suitable null hypothesis would be that the three curves have the same shape. The “lagging” by countries with high income inequality can be described as a lower  $m$ -value. The Richards model tells us that this is more than a lagging: the model is predicting lower asymptotic values for the total number of users, by several orders of magnitude. It also tells us that the diffusion of internet use in countries with higher income inequality is characterized by a maximum rate of growth per unit time that is an order of magnitude lower than those of the other two

Table 3

Richards model parameters for average diffusion of internet use, by income inequalities. Data from Noh and Yoo [22].

Income inequality	$m$	$W^\infty$	$T^*$	$k$	Converged?	Pr>F	RMSE	Data Standard Deviation
High income inequality	1.46	115	4.11	0.2057	Yes	0.0001	4.22	35.20
Medium income inequality	5.82	1,710,652	30	0.2100	Yes	0.0001	11.75	82.06
Low income inequality	6.07	10,613,526	35	0.2100	Yes	0.0001	7.24	95.71

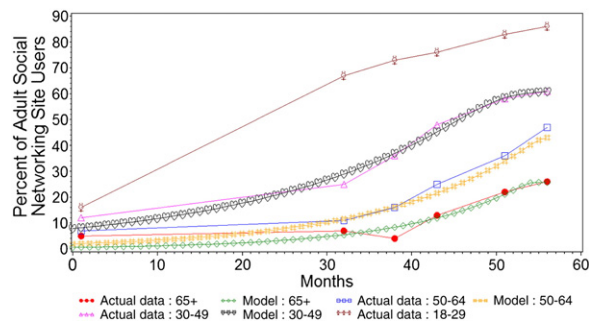


Fig. 7. Use of social networking websites, by age groups. Data from Madden [23].

Table 4

Richards model parameters for average diffusion of social networking websites, by age group. Data from Madden [23].

Age group	m	$W^\infty$	$T^*$	k	Converged?	Pr>F	RMSE	Data standard deviation
65+	61.54	25.77	52.87	0.0654	No.	0.0594	18.55	9.28
50–64	37.31	43.31	53.55	0.0499	No.	0.0248	33.54	15.46
30–49	13.12	61.09	45.28	0.0306	Yes.	0.0068	53.42	19.26
18–29	–	–	–	–	–	–	–	–

categories. Moreover, whereas the maximum growth rates differ, the maximum relative growth rates for all three categories do not differ or they do not differ by much. Relative growth rate is the growth rate relative to the size of the population.

A recent study [23] by the Pew Internet and American Life Project of Social Networking use, analyzing the use of social networking websites by age groups, illustrates the type of analysis that could be done if data were collected according to the requirements of the model, i.e., relatively dense data points at least up to the first inflection point. The model was unable to statistically significantly fit all the data, most likely due to its sparse quality (Fig. 7, Table 4). The fit for the age group 18–29 was particularly bad and is not presented here. Given those caveats, the Richards model allows quantification of the patterns: as age increases, m increases,  $W^\infty$  decreases,  $T^*$  increases, and k increases. The m-values from the Pew's study on social networking in the United States are much higher than the m-values from the internet diffusion data in Noh and Yoo [22]; but as might be expected, they are closer to the m-value for the countries with relatively low income inequality ( $m = 6.07$ ), which include the United States.

## 5. Conclusion

The Richards model accurately modeled technology diffusion data from several sources (Table 1, Fig. 3a–f). In all cases, regarding the model, the probability of the F ratio being as large or greater (under the null hypothesis) than the value that was obtained was  $P = 0.0001$ . It was demonstrated that the model is also useful in forecasting. Accurate parameter estimates could be obtained when the data was sparse but traced out the curve at least up to the third inflection point (concave down), and when the data was dense and traced out the curve up to the first inflection point (concave up). The Richards model parameter (m,  $W^\infty$ ,  $T^*$ , and k) phase space (or subsets thereof) is suggested as providing the basis for an alternative to Rogers' technology diffusion typology (Fig. 4), and as a means to facilitate both hypothesis testing and quantitative comparisons.

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**Yorgos Marinakis** has a B.A. in Mathematics from San Jose State University, a J.D. and a Ph.D. in the biological sciences from the University of New Mexico, and is currently working on an M.B.A. at the Anderson School of Management. He is a registered patent attorney and is admitted to the NM and Massachusetts State Bars. He is an editor of the *Journal of Sustainability*.