

## Chapter 09

# Network Theorems 網路電路定理



C-C Tsai Source: Circuit Analysis: Theory and Practice ©Delmar Cengage Learning



## Outline

- **Superposition Theory** 重疊定理
- **Thévenin's Theory** 戴維寧定理
- **Norton's Theory** 諾頓定理
- **Maximum Power Transfer**  
最大功率轉移
- **Millman's Theorem** 密爾門定理



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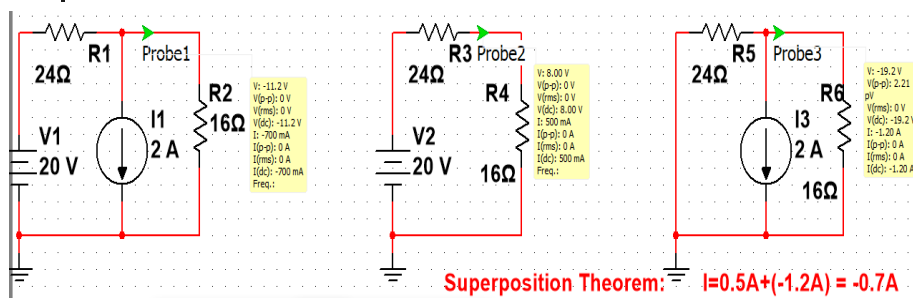
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# Superposition Theorem

## 重疊定理

- Total current through** or **voltage across** a resistor or branch: **Determine by adding effects due to each source acting independently**  
 一次只考量一個電源 (電壓源或電流源)
- Replace a voltage source with a short;**  
**Replace a current source with an open**  
 未考量者: 電壓源給予短路; 電流源給予開路
- Find results of branches using each source independently: **Algebraically combine results**  
 將各個電源的部分解而相加起來成為最後的解

## Example: Superposition Theorem



- Superposition Theorem does not apply to power:**  
**Due to Power is Not a linear quantity**  
 重疊定理不適用一次只考量各個電源之部分功率的總合，  
 因功率與電壓或電流源非線性關係

## Thévenin's Theorem

### 戴維寧定理

- Any linear bilateral network can be reduced to a **two-terminal circuit with a single voltage source** (Thévenin equivalent voltage,  $E_{Th}$ ) in series with a **single resistor** (Thévenin equivalent resistance,  $R_{Th}$ )  
任一雙端電路網路都可簡化為一戴維寧等效電壓源  $E_{Th}$  串聯一戴維寧等效電阻  $R_{Th}$

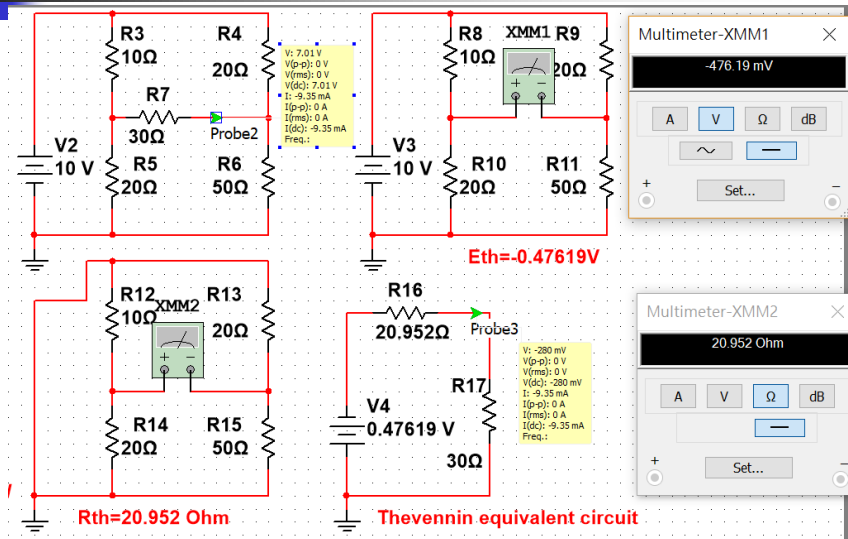
## Thévenin's Theorem

### 戴維寧定理

Steps to convert to a Thévenin circuit

- Remove the load from circuit** 暫移除負載電阻
- Replace voltage sources with shorts, current sources with opens to **determine Thévenin equivalent resistance** as seen by open circuit. 將電壓源給予短路及電流源給予開路而求得兩端之戴維寧等效電阻  $R_{Th}$
- Calculate voltage across open (If there is more than one source, Superposition theorem could be used) to **determine Thévenin equivalent voltage** as seen by open circuit. 求兩端開路之戴維寧等效電壓源  $E_{Th}$
- Draw Thévenin equivalent circuit, including the **load**. 將戴維寧等效電壓源  $E_{Th}$  串聯一戴維寧等效電阻  $R_{Th}$  再將負載電阻接至兩端

## Example: Thévenin's Theorem



## Norton's Theorem

### 諾頓定理 (類似於戴維寧定理)

- Any linear bilateral network can be reduced to a **two-terminal circuit with a single current source (Norton equivalent current  $I_N$ ) in parallel with a single resistor (equivalent resistance  $R_N = R_{Th}$ )**

任一雙端電路網路都可簡化為一諾頓等效電流源  $I_N$

並聯一諾頓等效電阻  $R_N$

## Norton's Theorem

### 諾頓定理

Steps to convert to a Norton's circuit

- **Remove the load from circuit** 暫移除負載電阻
- Replace voltage sources with shorts, current sources with opens to **determine Norton equivalent resistance** as seen by open circuit. 將電壓源給予短路及電流源給予開路而求得兩端之戴維寧等效電阻  $R_N$
- Determine current that **would flow through a short place between two terminals** (For multiple sources, Superposition theorem could be used). This is the **Norton's equivalent current**. 求兩端短路之諾頓等效電流源  $I_N$
- Draw Norton's equivalent circuit, including the **load**. 將諾頓等效電流源  $I_N$  並聯諾頓等效電阻  $R_N$  再將負載電阻接至兩端



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## Maximum Power Transfer

### 最大功率轉移

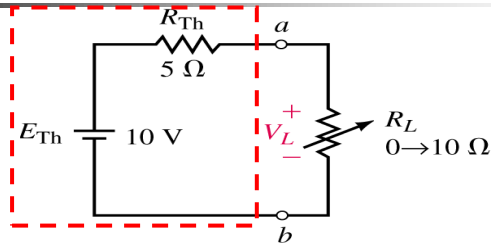
- **A load resistor will receive maximum power** from a circuit when its resistance is the same as the Thévenin (or Norton) equivalent resistance 當負載電阻  $R_L$  等於戴維寧(或諾頓)等效電阻  $R_{TH}$ (或  $R_N$ ) 則可得到最大功率轉移



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## Maximum Power Transfer



$$R_L = 1\Omega, I = E/(R_{th} + R_L) = 10/(5 + 1) = 5/3, P_{RL} = I^2 R_L = (5/3)^2 \times 1 = 2.778 \text{ W}$$

$$R_L = 3\Omega, I = E/(R_{th} + R_L) = 10/(5 + 3) = 5/4, P_{RL} = I^2 R_L = (5/4)^2 \times 3 = 4.688 \text{ W}$$

$$R_L = 4\Omega, I = E/(R_{th} + R_L) = 10/(5 + 4) = 10/9, P_{RL} = I^2 R_L = (10/9)^2 \times 4 = 4.938 \text{ W}$$

$$\mathbf{R_L = 5\Omega, I = E/(R_{th} + R_L) = 10/(5 + 5) = 1, P_{RL} = I^2 R_L = (1)^2 \times 5 = 5 \text{ W}}$$

$$R_L = 6\Omega, I = E/(R_{th} + R_L) = 10/(5 + 6) = 10/11, P_{RL} = I^2 R_L = (10/11)^2 \times 6 = 4.958 \text{ W}$$

$$R_L = 7\Omega, I = E/(R_{th} + R_L) = 10/(5 + 7) = 5/6, P_{RL} = I^2 R_L = (5/6)^2 \times 7 = 4.861 \text{ W}$$

$$R_L = 9\Omega, I = E/(R_{th} + R_L) = 10/(5 + 9) = 5/7, P_{RL} = I^2 R_L = (5/7)^2 \times 9 = 4.592 \text{ W}$$

$$R_L = 10\Omega, I = E/(R_{th} + R_L) = 10/(5 + 10) = 2/3, P_{RL} = I^2 R_L = (2/3)^2 \times 10 = 4.444 \text{ W}$$

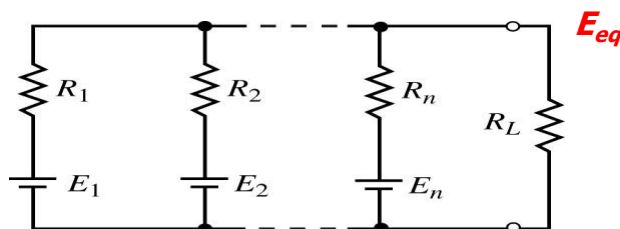


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## Millman's Theorem

### 密爾門定理

- Used to simplify circuits that have **Several parallel-connected branches containing a voltage source and series resistance** 適用於幾個電壓源(含串聯電阻)的分支並聯，求等效電壓  $E_{eq}$



$$E_{eq} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \dots + \frac{E_n}{R_n} + \frac{0}{R_L}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} + \frac{1}{R_L}}$$



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## Basic concept for Millman's Theorem

- First, convert voltage sources into current sources
- Second, **the equivalent current  $I_{eq}$**  is multiplied to the **equivalent resistance  $R_{eq}$**  to be the **equivalent voltage  $E_{eq}$** , i.e.,  $E_{eq} = I_{eq} * R_{eq}$

基本觀念: 首先轉換電壓源為電流源, 再將等效電流  $I_{eq}$  乘以等效電阻  $R_{eq}$  而得到等效電壓  $E_{eq}$

## Kernel abilities

1. Can use Superposition Theory for solving the unknown voltage and current of a circuit.
2. Can apply Thévenin's Theory for solving the unknown voltage and current of a circuit.
3. Can apply Norton's Theory for solving the unknown voltage and current of a circuit.
4. Can use Maximum Power Transfer for solving the output power.
5. Can recognize Millman's Theory for solving unknown voltage and current of a circuit.