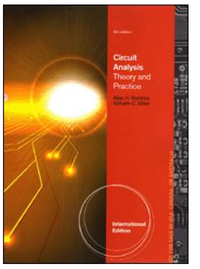


Chapter 14

Inductive Transients

Source: Circuit Analysis: Theory and Practice ©Delmar Cengage Learning

Transients

- Voltages and currents during a transitional interval
  - Referred to as transient behavior of the circuit
- **Capacitive circuit**
  - Voltages and currents undergo transitional phase
  - **Capacitor charges and discharges**
- **Inductive circuit**
  - Transitional phase occurs as the magnetic field builds and collapses

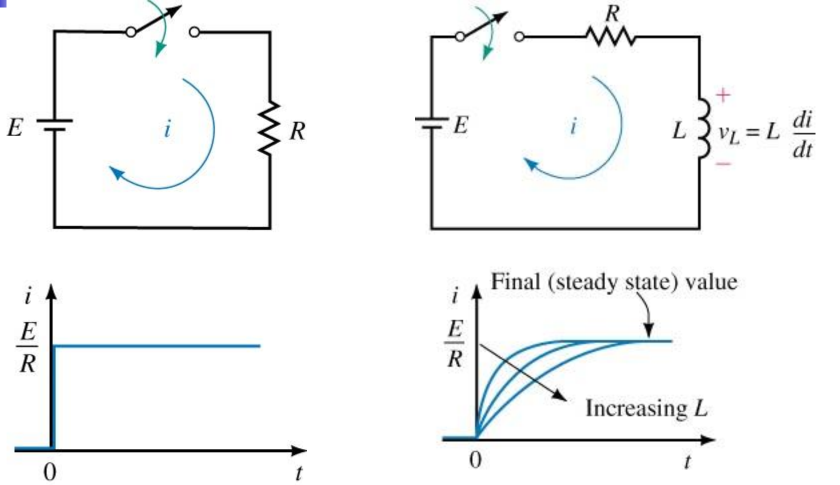
Voltage Across an Inductor

- Induced voltage across an inductor is proportional to rate of change of current

$$v_L = L\frac{\Delta i}{\Delta t}$$

- If inductor current could change instantaneously
  - Its rate of change would be infinite
  - Would cause infinite voltage
- Infinite voltage is not possible
  - **Inductor current cannot change instantaneously**
  - **It cannot jump from one value to another**, but must be **continuous at all times**

Transient due to Inductor

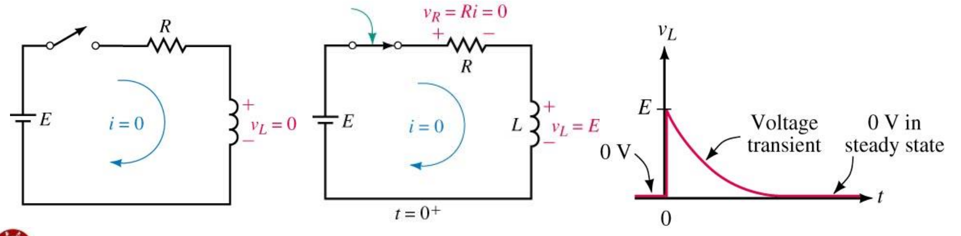


(a) No transient occurs in a purely resistive circuit.

(b) Adding inductance causes a transient to appear. R is held constant here.

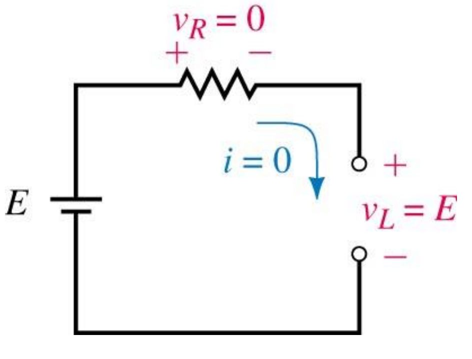
Inductor Voltage

- Immediately after closing the switch on an *RL* circuit
  - Current is zero
  - Voltage across the resistor is zero
- Voltage across resistor is zero
  - Voltage across inductor is source voltage
- Inductor voltage will then exponentially decay to zero



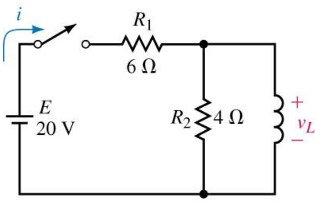
Open-Circuit Equivalent

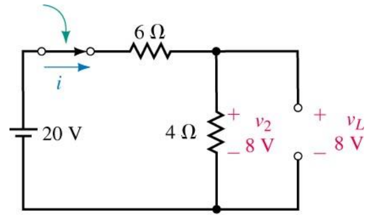
- After switch is closed (**t=0⁺**)
  - Inductor has voltage across it and no current through it
- Inductor with zero initial current looks like an open circuit at instant of switching

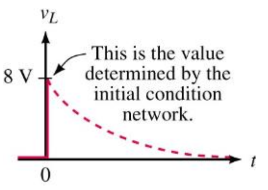


Initial Condition Circuits

- Voltages and currents in circuits immediately after switching **(t=0⁺)**
  - Determined from the open-circuit equivalent
- **By replacing inductors with opens**
  - We get initial condition circuit
- Initial condition networks
  - Yield voltages and currents only at switching

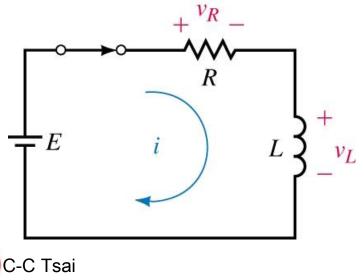






Circuit Current and Circuit Voltages

- **Current $i(t)$** in an RL circuit is an exponentially increasing function of time
- Current begins at zero and rises to a maximum value
- **Voltage across resistor V_R** is an exponentially increasing function of time
- **Voltage across inductor V_L** is an exponentially decreasing function of time



$$i(t) = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

$$v_R = E\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$v_L = E \bullet e^{-Rt/L}$$

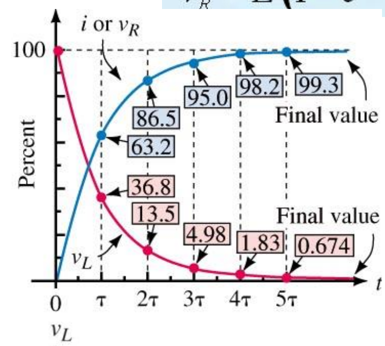
Time Constant

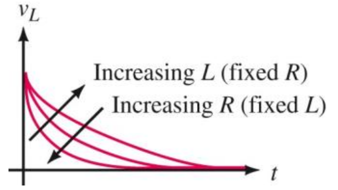
$$i = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$

$$v_L = E \cdot e^{-t/\tau}$$

$$v_R = E\left(1 - e^{-t/\tau}\right)$$

- $\tau = L/R$, units are seconds
- The larger the inductance
  - The longer the transient
- The larger the resistance
  - The shorter the transient
- As *R* increases
  - Circuit looks more and more resistive
  - If *R* is much greater than *L*, the circuit looks purely resistive



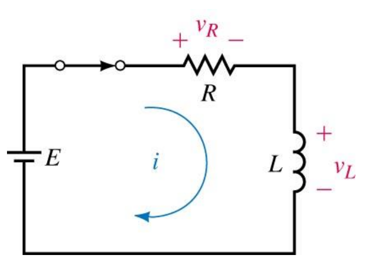


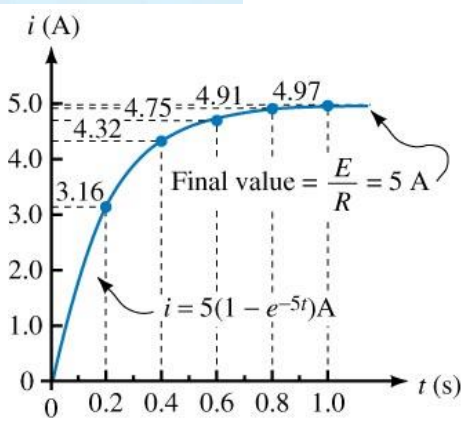
Example: RL Transients

- Given *E*=50V, *R*=10Ω, and *L*=2H, determine *i*(*t*).

$\tau = L/R = 0.2$s

$$i(t) = \frac{E}{R}\left(1 - e^{-Rt/L}\right) \quad \text{or} \quad i = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$



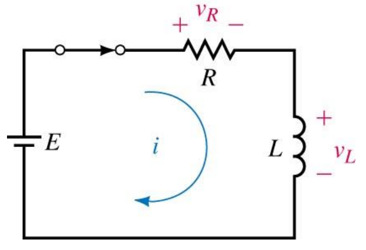


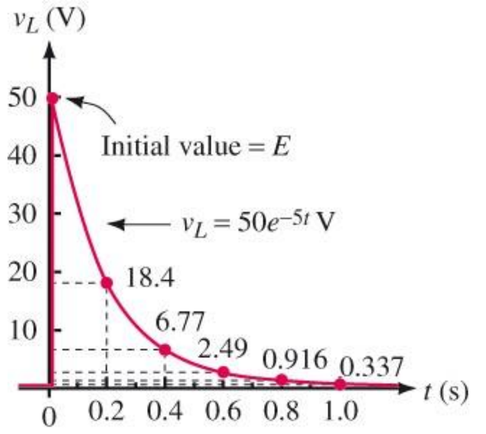
Example: RL Transients

- Given E=50V, R=10Ω, and L=2H, determine $i(t)$.

τ = L/R=0.2s

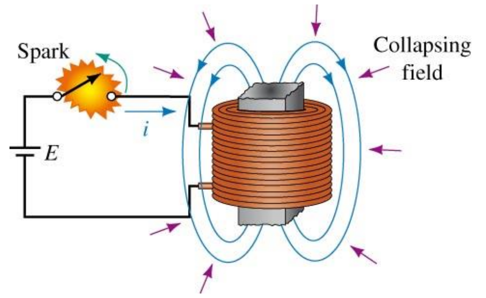
$$v_L = E \cdot e^{-t/\tau}$$





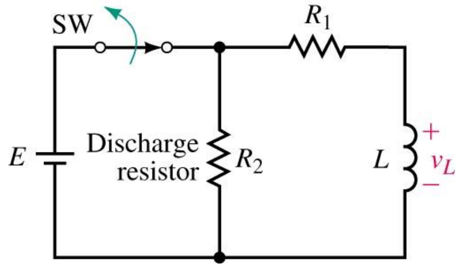
Interrupting Current in an Inductive Circuit

- When switch opens in an *RL* circuit
  - Energy is released in a short time
  - This may create a large voltage
  - Induced voltage is called an inductive kick
- Opening of inductive circuit may cause **voltage spikes of thousands of volts**

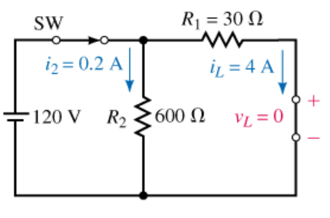


Interrupting a Circuit

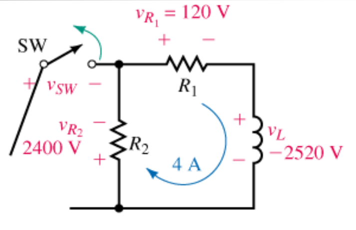
- Switch flashovers are generally undesirable
  - They can be controlled with proper engineering design
- These large voltages can be useful
  - Such as in **automotive ignition systems**
- It is not possible to completely analyze such a circuit
  - Resistance across the arc changes as the switch opens



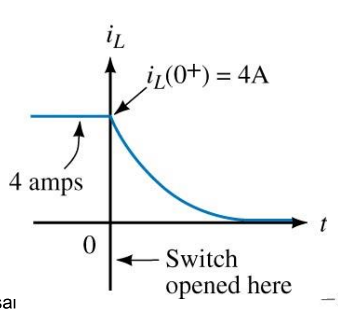
Example: Interrupting a Circuit

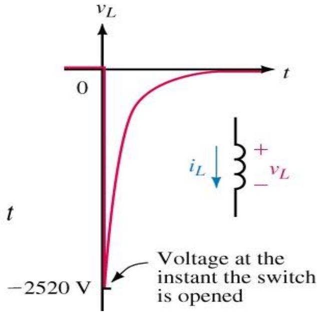


(a) Circuit just before the switch is opened



(b) Circuit just after SW is opened. Since coil voltage polarity is opposite to that shown, v_L is negative





Inductor Equivalent at Switching

- Current through an inductor
  - Same after switching as before switching
- <u>An inductance with an initial current</u>
  - Looks like a current source at instant of switching
- Its value is value of current at switching

(a) Current at switching (b) Current source equivalent

De-energizing Transients

- If an inductor has an initial current I_0, equation for **current** becomes

$$i = I_0 e^{-t/\tau'}$$

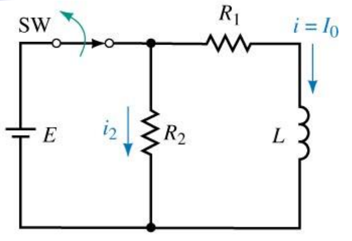
- $\tau' = L/R$. R equals total resistance in discharge path
- **Voltage across inductor** goes to zero as circuit de-energizes

$$v_L = V_0 e^{-t/\tau'}$$

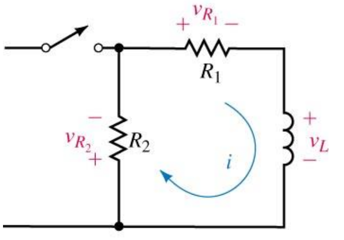
- **Voltage across any resistor** is product of current and that resistor. Voltage across each of resistors goes to zero

$$v_R = R \bullet I_0 e^{-t/\tau'}$$

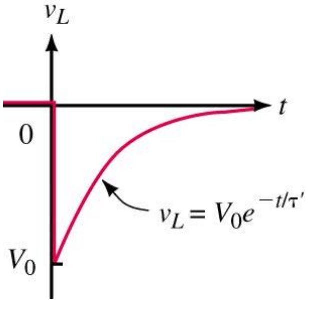
De-energizing Transients



(a) Immediately before the switch is opened

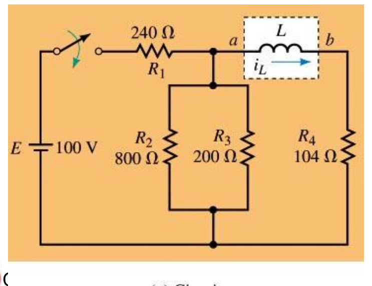


(b) Decay circuit

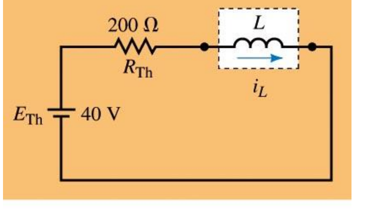


More Complex Circuits

- For complex circuits
  - Determine **Thévenin equivalent** circuit using inductor as the load
- R_{Th} is used to determine time constant
- $\tau = L/R_{Th}$
- E_{Th} is used as source voltage

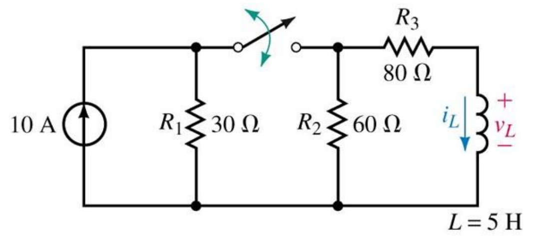


(a) Circuit

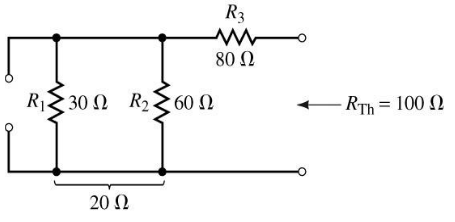


(b) Thévenin equivalent

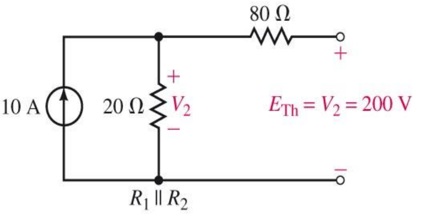
Example1: More Complex Circuits



Switch is closed



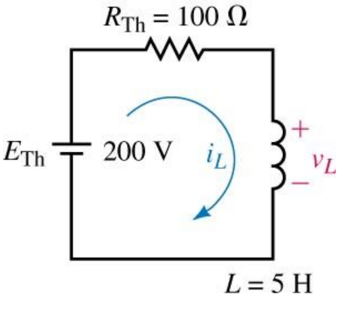
(a)



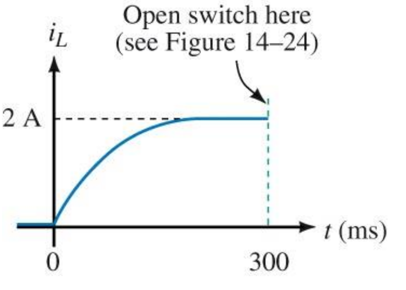
(b)

Example1: More Complex Circuits

Switch is closed (Cont'd)

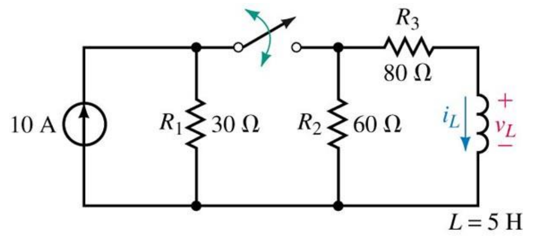


(a) Thévenin equivalent of Figure 14–24

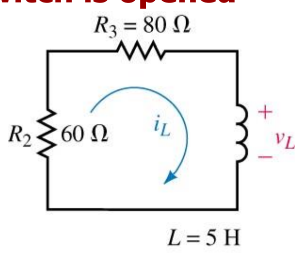


(b)

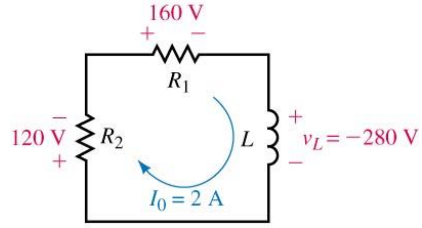
Example1: More Complex Circuits



Switch is opened

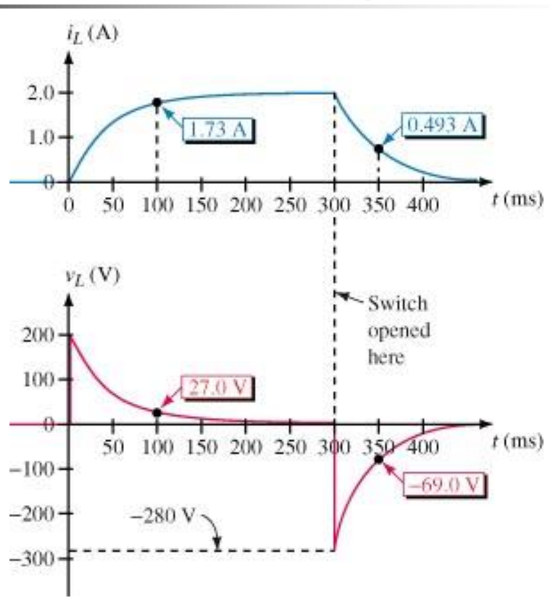


(a) Decay circuit

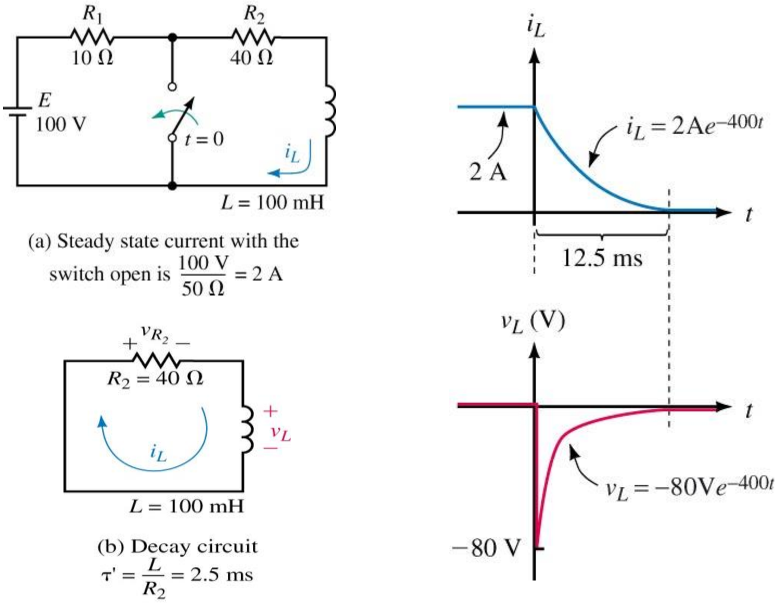


(b) As it looks immediately after the switch is opened. KVL yields $v_L = -280$ V

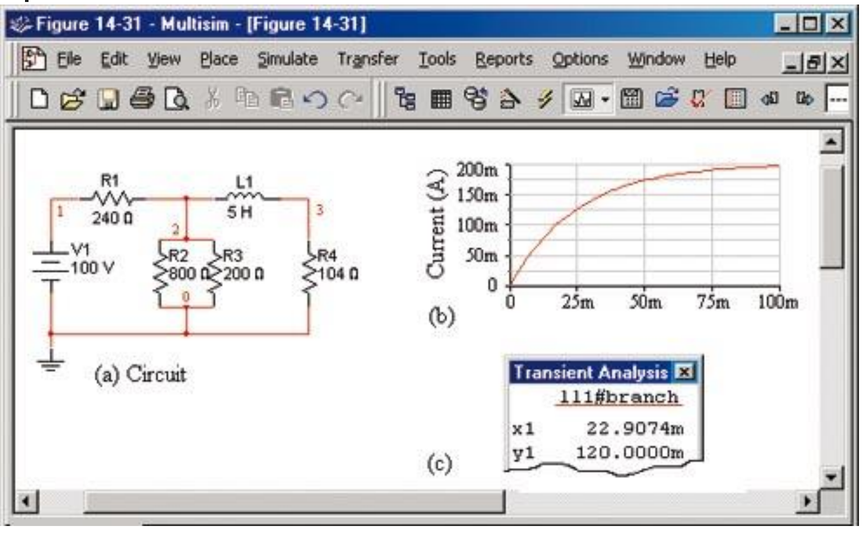
Example1: More Complex Circuits



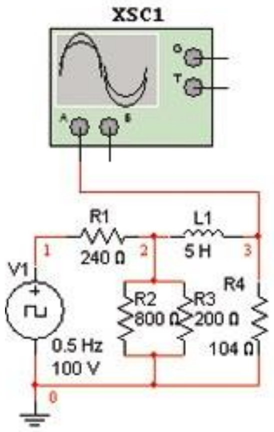
Example2: More Complex Circuits

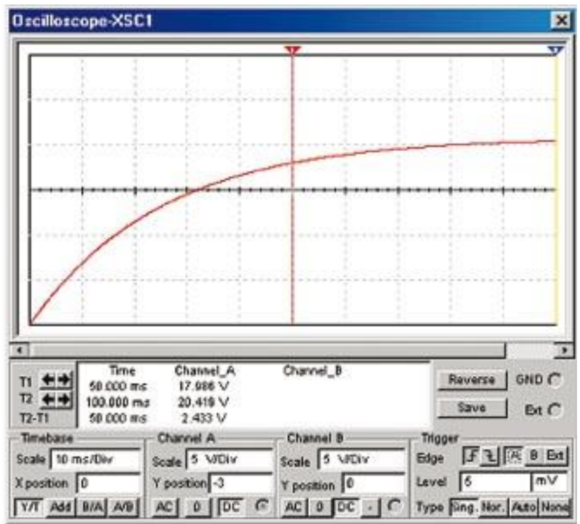


Transient Analysis Using Computers

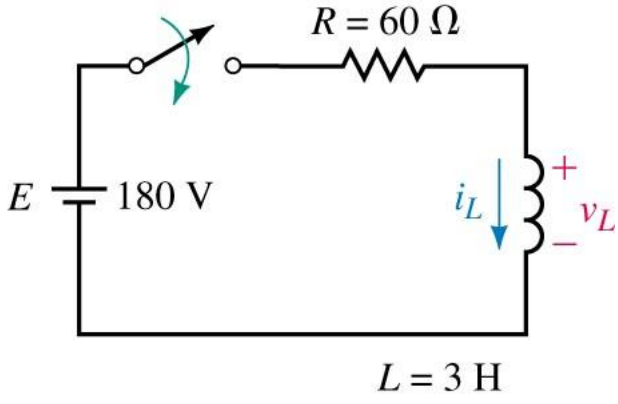


Transient Analysis Using Computers





Problem: Determine i_L and v_L



Problem: Determine i_L and v_L