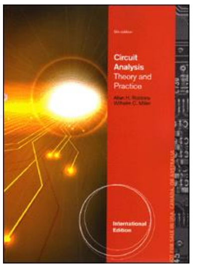
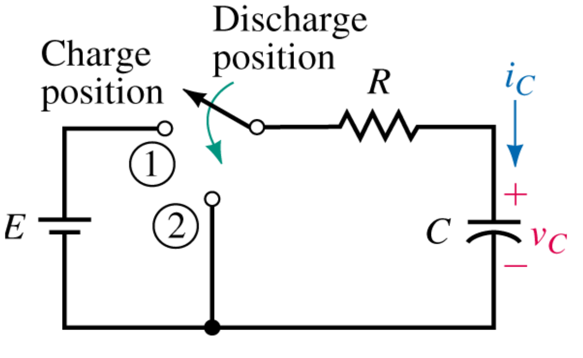


Chapter 11

Capacitors Charging, Discharging, Simple Waveshaping Circuits

Introduction

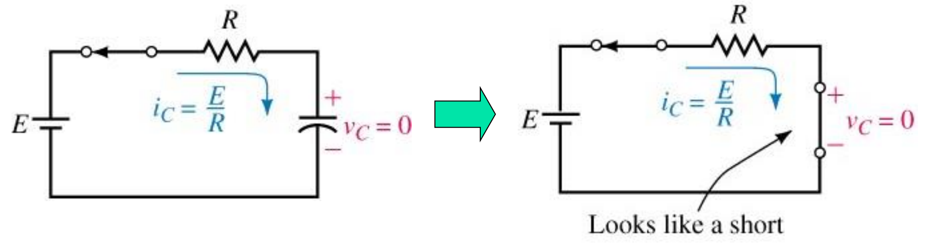
- When switch is closed at ①, **capacitor charging**
- When switch is closed at ②, **capacitor discharging**
- **Transient voltages and currents** result when circuit is switched



Capacitor Charging

When switch is closed at ①, beginning state

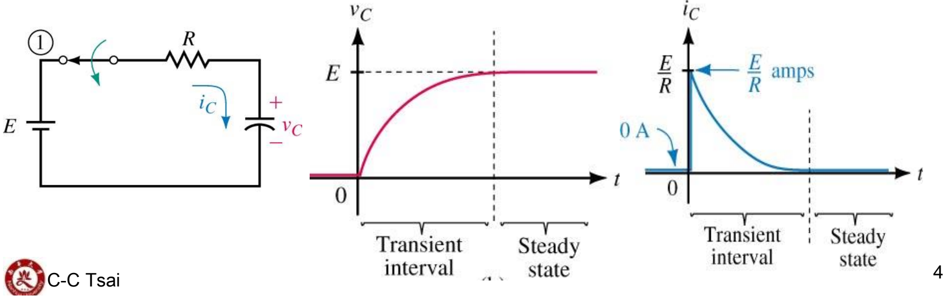
- Capacitor voltage cannot change instantaneously
- When switching, the capacitor looks like a short circuit
- **Capacitor voltage** begins at **zero**
- **Capacitor current** instantaneously jumps to ***E* / *R***



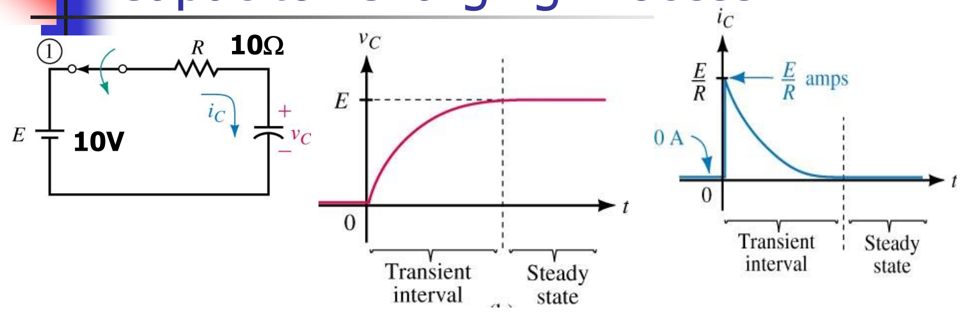
Capacitor Charging

When switch is closed at ①, transient state

- **Capacitor voltage** begins at zero and **exponentially increases to *E* volts**
- **Capacitor current** instantaneously jumps to *E* / *R* and **exponentially decays to zero**



Capacitor Charging Process



t=0s, v_c = 0V, i_c = (E - v_c)/10 = (10 - 0)/10 = 1A

t=1s, v_c = 6.3V, i_c = (E - v_c)/10 = (10 – 6.3)/10 = 0.37A

t=3s, v_c = 9.5V, i_c = (E - v_c)/10 = (10 – 9.5)/10 = 0.05A

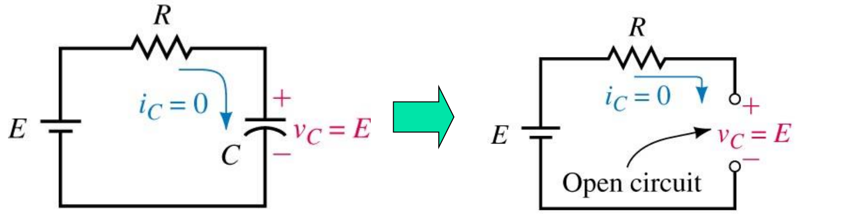
t=5s, v_c = 10V, i_c = (E - v_c)/10 = (10 – 10)/10 = 0A

t=6s, v_c = 10V, i_c = (E - v_c)/10 = (10 – 10)/10 = 0A

Steady State Conditions

When switch is closed at ①, steady state

- Capacitor voltage and current reach their final values and **stop changing**
- Capacitor has voltage across it, but no current flows through the circuit. **Capacitor looks like an open circuit.**

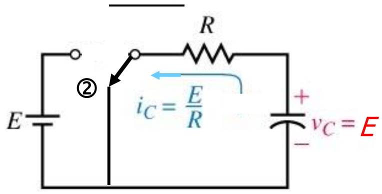


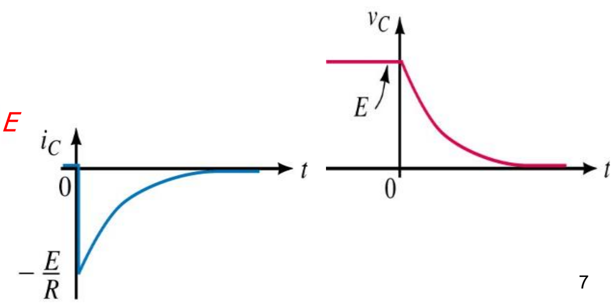
(a) $v_C = E$ and $i_C = 0$ (b) Equivalent circuit for the capacitor

Capacitor Discharging

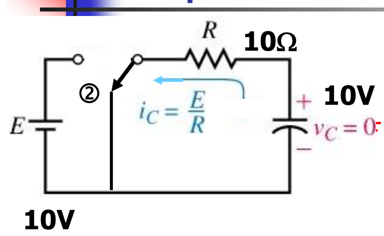
When switch is closed at ②, beginning state

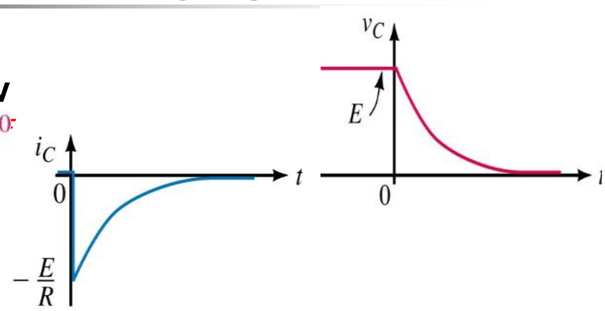
- **Capacitor voltage** has E volts across it when it begins to discharge
- **Capacitor current** will instantly jump to $-E/R$
- Both voltage and current will **decay exponentially to zero**





Capacitor Discharging Process





t=0s, v_c = 10V, i_c = -(v_c)/10 = -(10)/10 = -1A

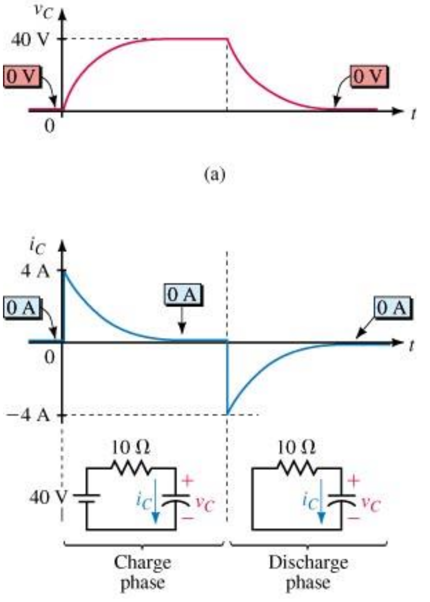
t=1s, v_c = 3.7V, i_c = -(v_c)/10 = -(3.3)/10 = -0.37A

t=3s, v_c = 0.5V, i_c = -(v_c)/10 = -(0.5)/10 = -0.05A

t=5s, v_c = 0V, i_c = -(v_c)/10 = (0)/10 = 0A

t=6s, v_c = 0V, i_c = -(v_c)/10 = (0)/10 = 0A

Example: Capacitor Charging/Discharge



Capacitor Charging Equations

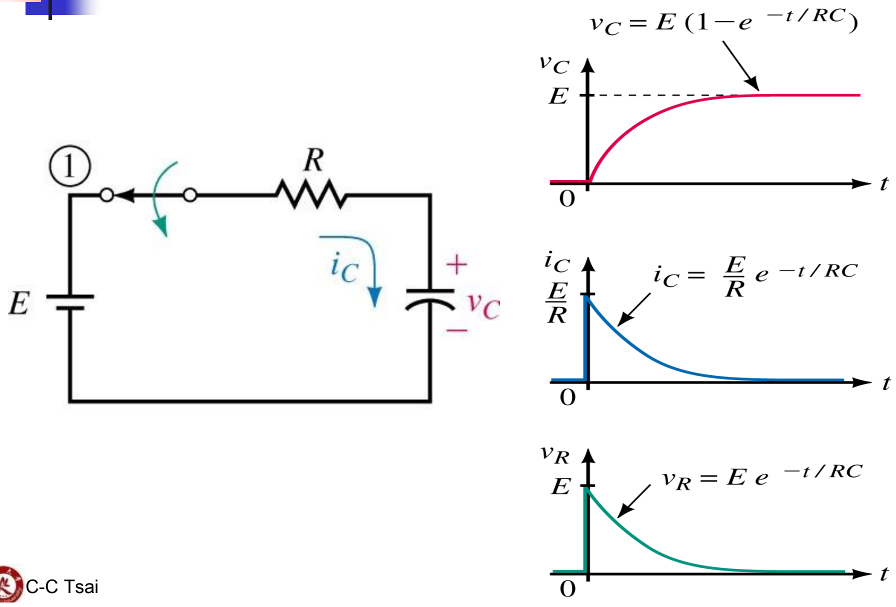
- Voltages and currents in a charging circuit do not change instantaneously
- These changes over time are **exponential** changes
- **The voltage** across the capacitor as a function of time is

$$v_C = E(1 - e^{-t/RC})$$

- **The current** through the capacitor as a function of time is

$$i_C = \frac{E}{R} e^{-t/RC}$$

Capacitor Charging Equations



The Time Constant

- Rate at which a capacitor charges **depends on product of *R* and *C***
- Product known as **time constant**, $\tau = RC$
  τ (Greek letter tau) has units of seconds
- Length of time that a transient lasts depends on **exponential function $e^{-t/\tau}$**.
- As *t* increases, the function decreases. When the *t* reaches infinity, the function decays to zero
- For all practical purposes, transients can be considered to last for only five time constants

Exponential Functions

x	e^{-x}	$1 - e^{-x}$
0	1	0
1	0.3679	0.6321
2	0.1353	0.8647
3	0.0498	0.9502
4	0.0183	0.9817
5	0.0067	0.9933

The Time Constant

The functions $e^{-t/\tau}$ and $1-e^{-t/\tau}$

$$v_C = E(1 - e^{-t/RC})$$

t=0RC=0τ, e^{-0} = 1 , $E(1-e^{-0})$= 0

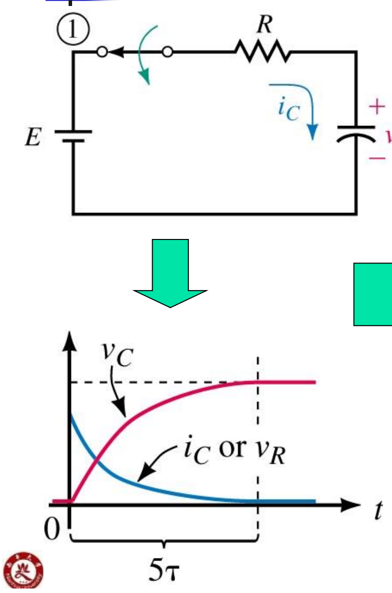
t=1RC=1τ, e^{-1} = 0.368 , $E(1-e^{-1})$= 0.632×E

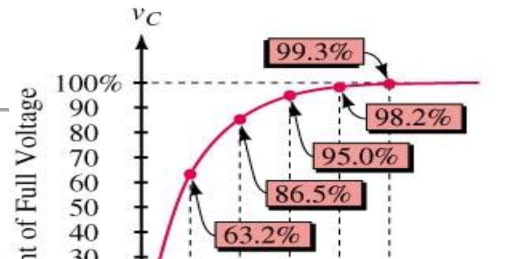
t=2RC=2τ, e^{-2} = 0.135 , $E(1-e^{-2})$= 0.865×E

t=3RC=3τ, e^{-3} = 0.050 , $E(1-e^{-3})$= 0.950×E

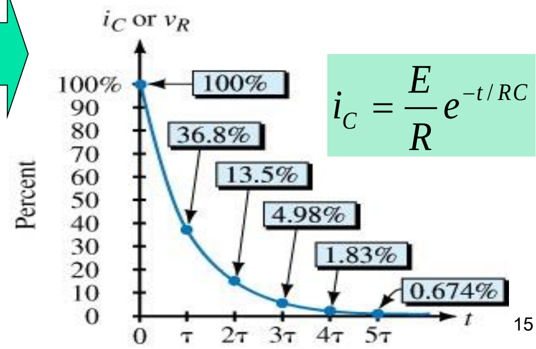
t=4RC=4τ, e^{-4} = 0.018 , $E(1-e^{-4})$= 0.982×E

t≥5RC=5τ, e^{-5} = 0.007 , $E(1-e^{-5})$= 0.993×E



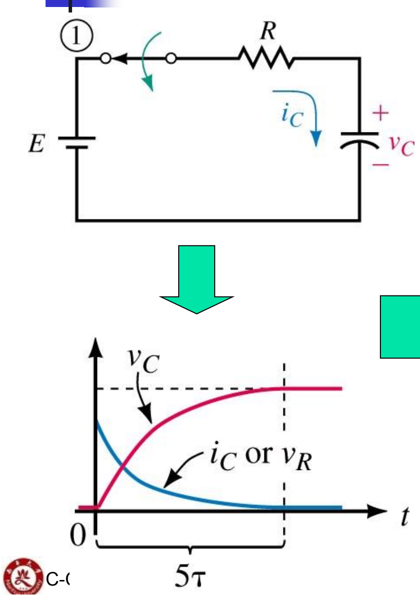


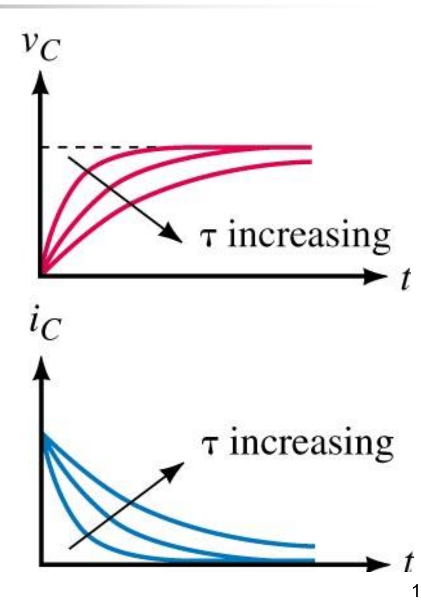
$$v_C = E(1 - e^{-t/RC})$$



$$i_C = \frac{E}{R} e^{-t/RC}$$

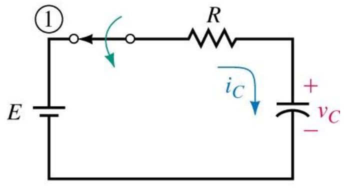
The Time Constant

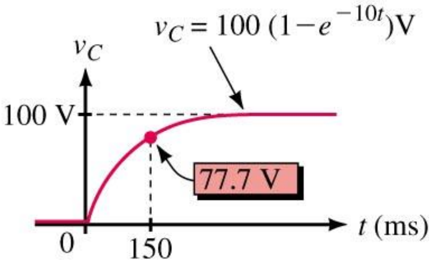


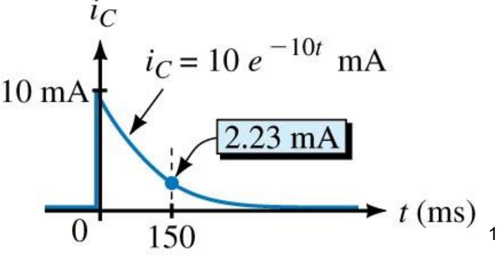


Example1: Capacitor Charging

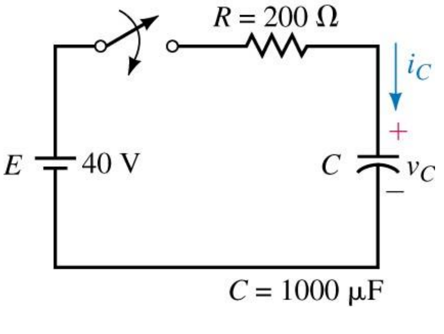
A RC circuit with *E*=100V, *R*=10KΩ, and *C*=10μF





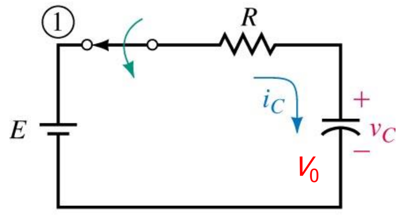


Example2: Capacitor Charging



Capacitor with an Initial Voltage

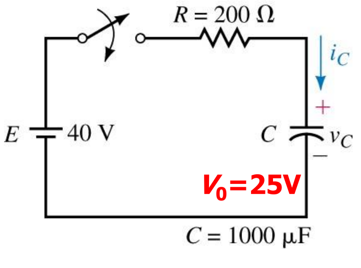
- Voltage denoted as V_0
  - Capacitor has a voltage on it
- Voltage and current in a circuit will be affected by initial voltage

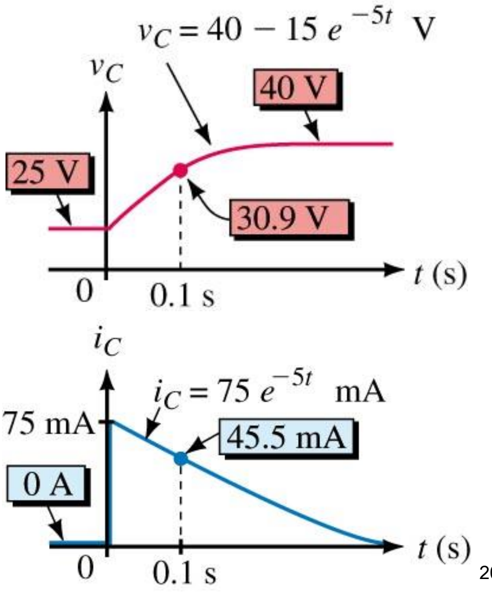


$$v_C = E + (V_0 - E)e^{-t/\tau}$$

$$i_C = \frac{E - V_0}{R} e^{-t/\tau}$$

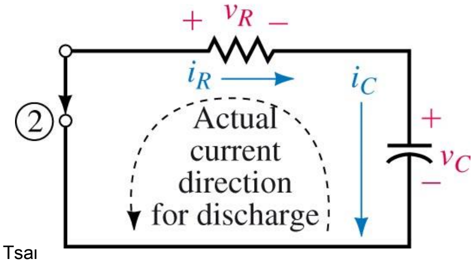
Example: Capacitor with an Initial Voltage





Capacitor Discharging Equations

- If a capacitor is **charged to voltage V_0** and **then discharged**.
- Current is negative because it flows opposite to reference direction
- Discharge transients last five time constants
- All voltages and currents are at zero when capacitor has fully discharged

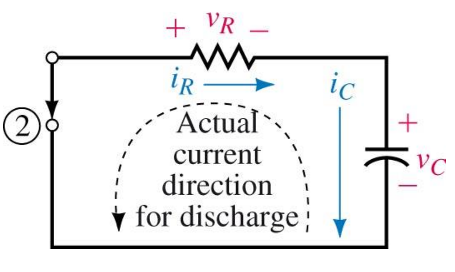


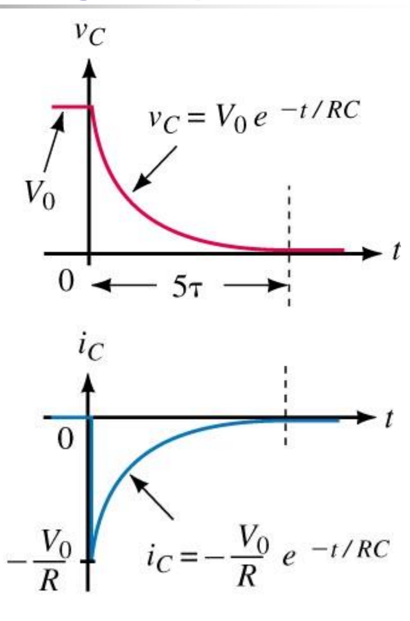
$$v_C = V_0 e^{-t/\tau}$$

$$v_R = -V_0 e^{-t/\tau}$$

$$i_C = -\frac{V_0}{R} e^{-t/\tau}$$

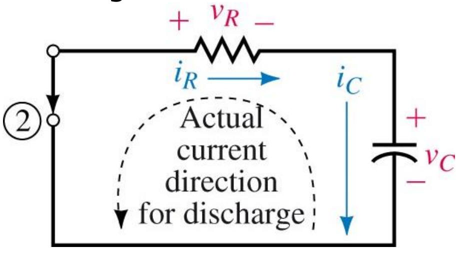
Capacitor Discharge Equations





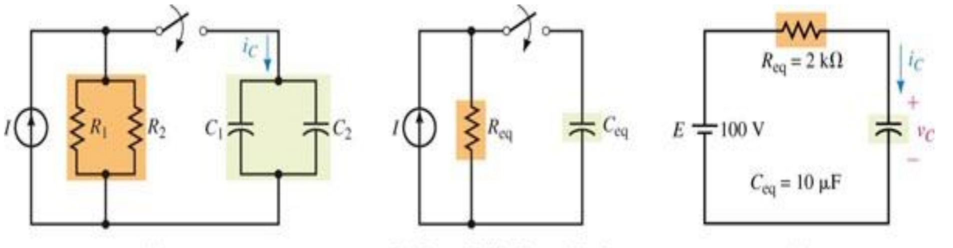
Example: Capacitor Discharge

A RC circuit with ***R*=5KΩ** and ***C*=25μF**, assume that *C* has charged to **100V**. Determine the discharge voltage and current.



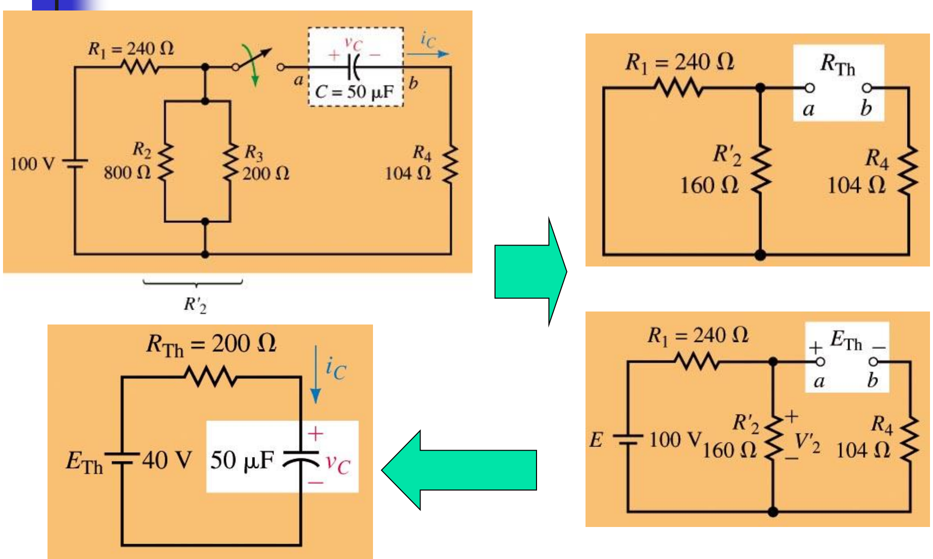
More Complex Circuits

- Remove capacitor as the load and determine **Thévenin equivalent circuit**
- Use R_{Th} to determine τ, $\tau = R_{Th} \cdot C$
- Use E_{Th} as the equivalent source voltage

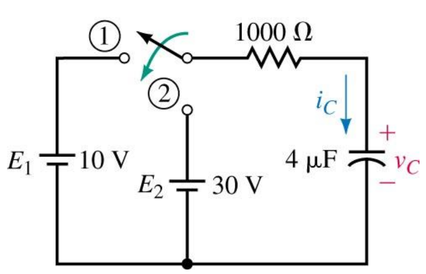


(a) (b) $R_{eq} = 2\ k\Omega$, $C_{eq} = 10\ \mu F$ (c)

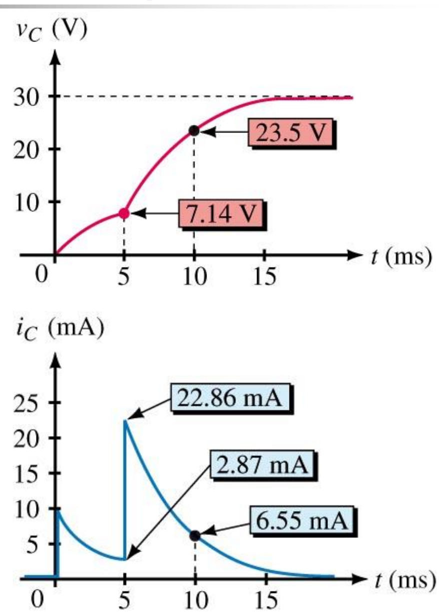
Example: More Complex Circuits



Example: More Complex Circuits

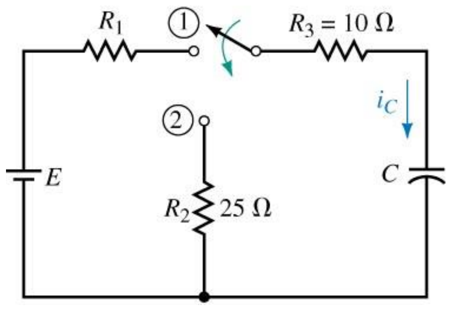


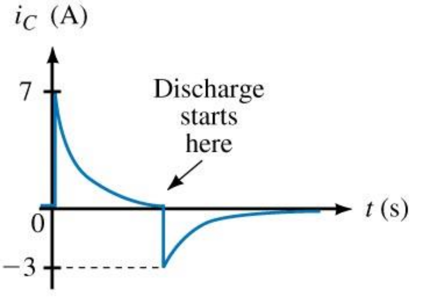
The switch is closed at ① for 5ms then closed at ② for 10ms.



Example: More Complex Circuits

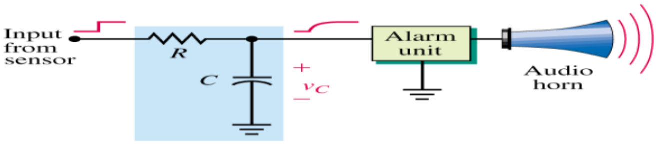
The capacitor takes 1.75ms to discharge as shown the waveform. Determine *E*, *R*$_1$, and *C*.



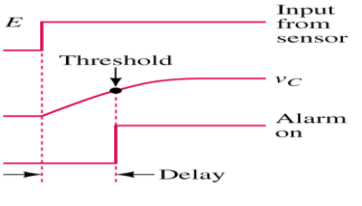


An *RC* Timing Applications

- *RC* circuits are used to **create delays for alarm, motor control, and timing applications**
- **Alarm unit shown contains a threshold detector**
  - When input to this detector exceeds a preset value, the alarm is turned on

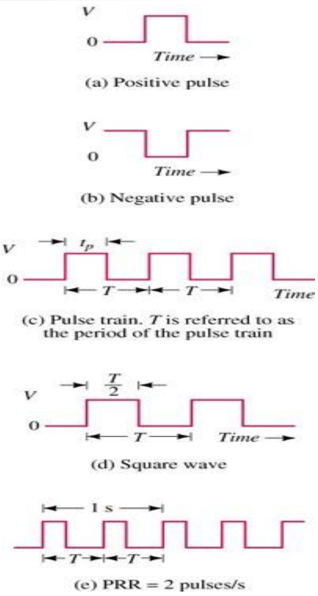


(a) Delay circuit



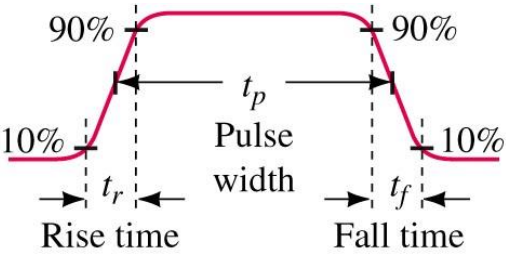
Pulse Response of *RC* Circuits

- **Pulse:** Voltage or current that changes from one level to another and back again
- **Periodic waveform**: Pulse train is a repetitive stream of pulses
- **Square wave**: Waveform's time high equals its time low
- **Frequency**: Number of pulses per second
- **Duty cycle**: Width of pulse compared to its period



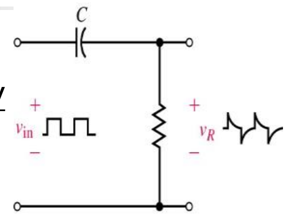
Pulse Response of *RC* Circuits

- Pulses have a rise and fall time
  - Because they do not rise and fall instantaneously
- **Rise** and **fall times** are measured between the 10% and 90% points

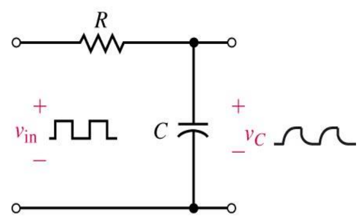


The Effect of Pulse Width

- If pulse width $t_p >> 5\tau$
  - Capacitor charges and discharges fully
  - This is a **differentiator circuit**

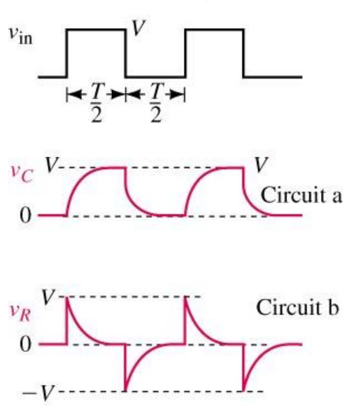


- If pulse width $t_p = 5\tau$
  - Capacitor fully charges and discharges during each pulse
- If the pulse width $t_p << 5\tau$
  - Capacitor **cannot** fully charge and discharge
  - This is an **integrator circuit**

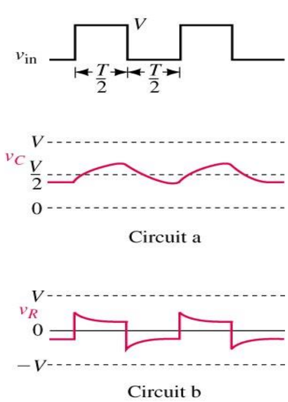


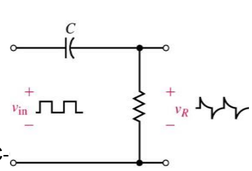
Simple Waveshaping Circuits

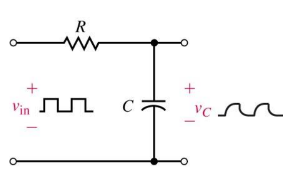
$T >> 5\tau$



$T << 5\tau$

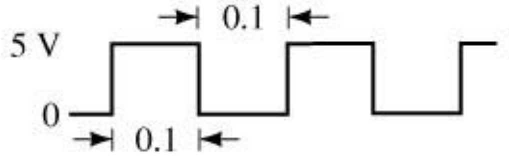




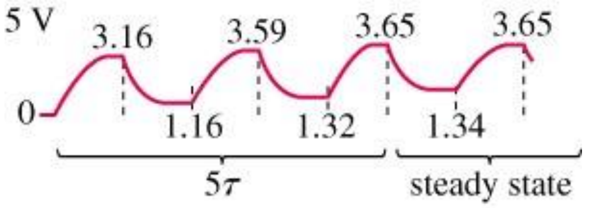


Simple Waveshaping Circuits

T **<< 5τ**



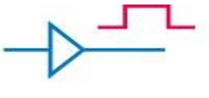
(a) Input waveform



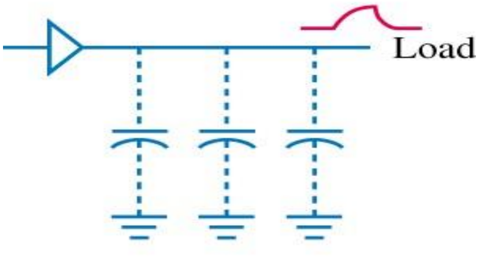
(b) Output voltage v_C

Capacitive Loading

- **In high-speed circuits, the capacitance can cause problems**

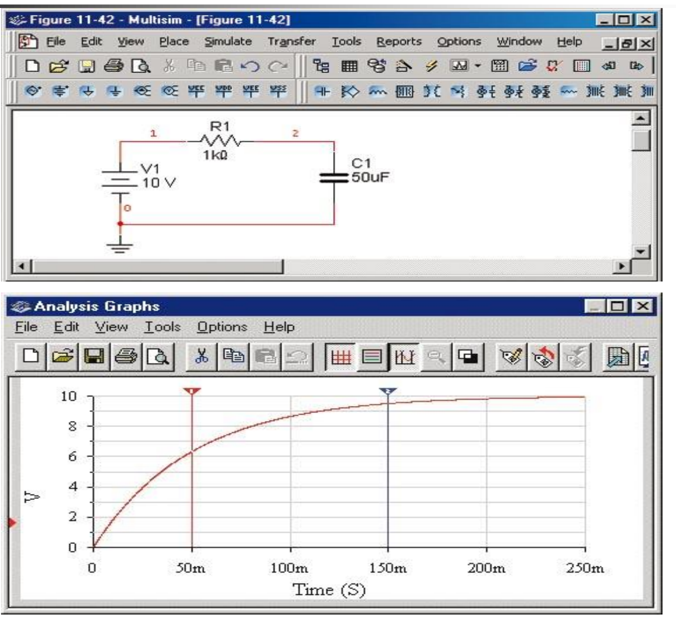


(a) Unloaded driver

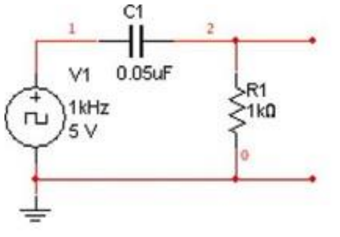


(b) Distorted signal

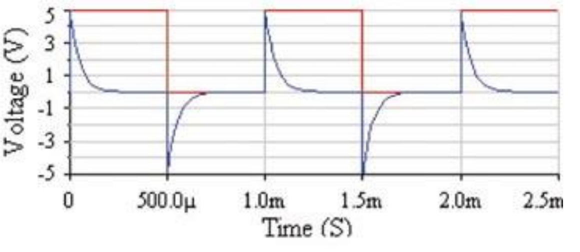
Transient Analysis Using Computers



Transient Analysis Using Multisim

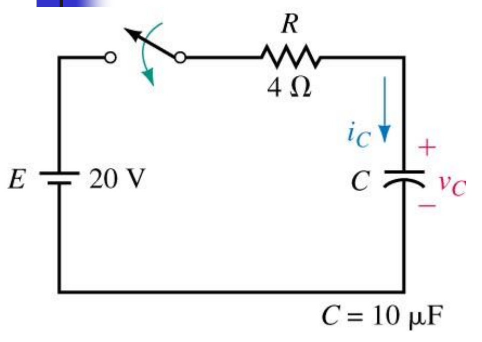


(a)



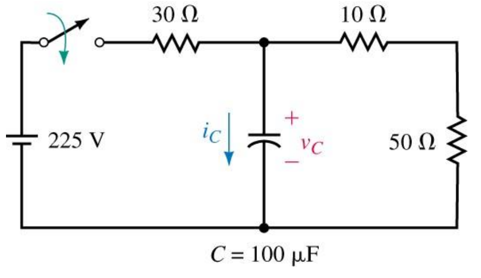
(b) The red waveform is the input and the blue one is the output

Problem: Draw the V_c waveform



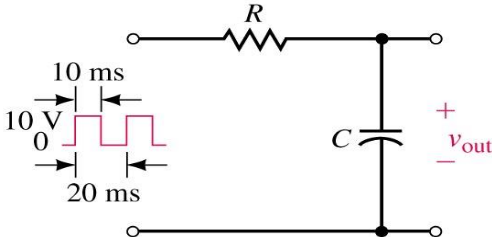
Problem:

Draw the V_c waveform after closing the switch for 15ms and opening the switch.



Problem:

Draw the V_{out} waveform if (a) R=2KΩ and C=0.1μF and (b) R=20KΩ and C=1μF.



Problem:

If Vc= 4.75V the alarm will be on, how long is required when the switch is closed to 5V.