Introduction

- When switch is closed at 1, 
  capacitor charging
- When switch is closed at 2, 
  capacitor discharging
- Transient voltages and currents result when circuit is switched
Capacitor Charging

When switch is closed at $\Theta$, beginning state

- Capacitor voltage cannot change instantaneously
- When switching, the capacitor looks like a short circuit
- **Capacitor voltage** begins at **zero**
- **Capacitor current** instantaneously jumps to $E/R$

![Diagram showing capacitor charging in the beginning state](image)

Looks like a short

---

Capacitor Charging

When switch is closed at $\Theta$, transient state

- **Capacitor voltage** begins at zero and **exponentially** increases to $E$ volts
- **Capacitor current** instantaneously jumps to $E/R$ and **exponentially decays to zero**

![Diagram showing capacitor charging in the transient state](image)
Capacitor Charging Process

\[ t=0s, \quad v_c = 0V, \quad i_c = (E - v_c)/10 = (10 - 0)/10 = 1A \]
\[ t=1s, \quad v_c = 6.3V, \quad i_c = (E - v_c)/10 = (10 - 6.3)/10 = 0.37A \]
\[ t=3s, \quad v_c = 9.5V, \quad i_c = (E - v_c)/10 = (10 - 9.5)/10 = 0.05A \]
\[ t=5s, \quad v_c = 10V, \quad i_c = (E - v_c)/10 = (10 - 10)/10 = 0A \]
\[ t=6s, \quad v_c = 10V, \quad i_c = (E - v_c)/10 = (10 - 10)/10 = 0A \]

Steady State Conditions

When switch is closed at \( \bigcirc \), steady state

- Capacitor voltage and current reach their final values and stop changing.
- Capacitor has voltage across it, but no current flows through the circuit. Capacitor looks like an open circuit.

(a) \( v_C = E \) and \( i_C = 0 \)
(b) Equivalent circuit for the capacitor
Capacitor Discharging

When switch is closed at $\Phi$, beginning state

- **Capacitor voltage** has $E$ volts across it when it begins to discharge
- **Capacitor current** will instantly jump to $-\frac{E}{R}$
- Both voltage and current will decay exponentially to zero

![Diagram of Capacitor Discharging]

Capacitor Discharging Process

$t=0s$, $v_c = 10V$, $i_c = -(v_c)/10 = -(10)/10 = -1A$
$t=1s$, $v_c = 3.7V$, $i_c = -(v_c)/10 = -(3.3)/10 = -0.37A$
$t=3s$, $v_c = 0.5V$, $i_c = -(v_c)/10 = -(0.5)/10 = -0.05A$
$t=5s$, $v_c = 0V$, $i_c = -(v_c)/10 = (0)/10 = 0A$
$t=6s$, $v_c = 0V$, $i_c = -(v_c)/10 = (0)/10 = 0A
Example: Capacitor Charging/Discharge

Capacitor Charging Equations

- Voltages and currents in a charging circuit do not change instantaneously.
- These changes over time are exponential changes.
- **The voltage** across the capacitor as a function of time is 
  \[ v_C = E(1 - e^{-t/RC}) \]
- **The current** through the capacitor as a function of time is 
  \[ i_C = \frac{E}{R}e^{-t/RC} \]
Capacitor Charging Equations

\[ v_C = E \left(1 - e^{-t/RC}\right) \]

The Time Constant

- Rate at which a capacitor charges depends on product of \( R \) and \( C \)
- Product known as **time constant**, \( \tau = RC \)
  - \( \tau \) (Greek letter tau) has units of seconds
- Length of time that a transient lasts depends on exponential function \( e^{-t/\tau} \).
- As \( t \) increases, the function decreases. When the \( t \) reaches infinity, the function decays to zero
- For all practical purposes, transients can be considered to last for only five time constants
Exponential Functions

<table>
<thead>
<tr>
<th>x</th>
<th>$e^{-x}$</th>
<th>$1 - e^{-x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.3679</td>
<td>0.6321</td>
</tr>
<tr>
<td>2</td>
<td>0.1353</td>
<td>0.8647</td>
</tr>
<tr>
<td>3</td>
<td>0.0498</td>
<td>0.9502</td>
</tr>
<tr>
<td>4</td>
<td>0.0183</td>
<td>0.9817</td>
</tr>
<tr>
<td>5</td>
<td>0.0067</td>
<td>0.9933</td>
</tr>
</tbody>
</table>

The Time Constant

The functions $e^{-t/\tau}$ and $1-e^{-t/\tau}$

$$v_C = E (1 - e^{-t/RC})$$

$t = 0$ $RC = 0\tau$, $e^{-0} = 1$, $E (1 - e^{-0}) = 0$

$t = 1$ $RC = 1\tau$, $e^{-1} = 0.368$, $E (1 - e^{-1}) = 0.632 \times E$

$t = 2$ $RC = 2\tau$, $e^{-2} = 0.135$, $E (1 - e^{-2}) = 0.865 \times E$

$t = 3$ $RC = 3\tau$, $e^{-3} = 0.050$, $E (1 - e^{-3}) = 0.950 \times E$

$t = 4$ $RC = 4\tau$, $e^{-4} = 0.018$, $E (1 - e^{-4}) = 0.982 \times E$

$t \geq 5$ $RC = 5\tau$, $e^{-5} = 0.007$, $E (1 - e^{-5}) = 0.993 \times E$
The Time Constant

\[ V_C = E (1 - e^{-t/RC}) \]

\[ i_C = \frac{E}{R} e^{-t/RC} \]

\[ \tau = RC \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 5 \]

\[ 6 \]

\[ 7 \]

\[ 8 \]

\[ 9 \]

\[ 10 \]

\[ 11 \]

\[ 12 \]

\[ 13 \]

\[ 14 \]

\[ 15 \]

\[ 16 \]
Example 1: Capacitor Charging

A RC circuit with $E=100\text{V}$, $R=10\text{K}\Omega$, and $C=10\mu\text{F}$

Example 2: Capacitor Charging

A RC circuit with $E=40\text{V}$, $R=200\Omega$, and $C=1000\mu\text{F}$
Capacitor with an Initial Voltage

- Voltage denoted as $V_0$
  - Capacitor has a voltage on it
- Voltage and current in a circuit will be affected by initial voltage

$$v_C = E + (V_0 - E)e^{-t/\tau}$$

$$i_C = \frac{E - V_0}{R} e^{-t/\tau}$$

Example: Capacitor with an Initial Voltage

$V_0 = 25\text{V}$

$$v_C = 40 - 15 e^{-5t} \text{ V}$$

$$i_C = 75 e^{-5t} \text{ mA}$$
Capacitor Discharging Equations

- If a capacitor is charged to voltage $V_0$ and then discharged.
  - Current is negative because it flows opposite to reference direction
  - Discharge transients last five time constants
  - All voltages and currents are at zero when capacitor has fully discharged

$$v_C = V_0 e^{-t/\tau}$$
$$v_R = -V_0 e^{-t/\tau}$$
$$i_C = -\frac{V_0}{R} e^{-t/\tau}$$
Example: Capacitor Discharge

A RC circuit with $R = 5 \text{K}\Omega$ and $C = 25 \mu\text{F}$, assume that $C$ has charged to 100V. Determine the discharge voltage and current.

More Complex Circuits

- Remove capacitor as the load and determine **Thévenin equivalent circuit**
- Use $R_{\text{Th}}$ to determine $\tau$, $\tau = R_{\text{Th}} \cdot C$
- Use $E_{\text{Th}}$ as the equivalent source voltage
Example: More Complex Circuits

The switch is closed at ① for 5ms then closed at ② for 10ms.
Example: More Complex Circuits

The capacitor takes 1.75ms to discharge as shown the waveform. Determine $E$, $R_1$, and $C$.

![Circuit Diagram]

An RC Timing Applications

- RC circuits are used to create delays for alarm, motor control, and timing applications
- Alarm unit shown contains a threshold detector
  - When input to this detector exceeds a preset value, the alarm is turned on
Pulse Response of RC Circuits

- **Pulse**: Voltage or current that changes from one level to another and back again
- **Periodic waveform**: Pulse train is a repetitive stream of pulses
- **Square wave**: Waveform’s time high equals its time low
- **Frequency**: Number of pulses per second
- **Duty cycle**: Width of pulse compared to its period

Pulse Response of RC Circuits

- Pulses have a rise and fall time
  - Because they do not rise and fall instantaneously
- **Rise** and **fall times** are measured between the 10% and 90% points
The Effect of Pulse Width

- If pulse width $t_p >> 5\tau$
  - Capacitor charges and discharges fully
  - This is a differentiator circuit

- If pulse width $t_p = 5\tau$
  - Capacitor fully charges and discharges during each pulse

- If the pulse width $t_p << 5\tau$
  - Capacitor cannot fully charge and discharge
  - This is an integrator circuit

Simple Waveshaping Circuits

$T >> 5\tau$

$T << 5\tau$
Simple Waveshaping Circuits

$T \ll 5\tau$

(a) Input waveform

(b) Output voltage $v_C$

Capacitive Loading

In high-speed circuits, the capacitance can cause problems

(a) Unloaded driver

(b) Distorted signal
Transient Analysis Using Computers

(a) Transient Analysis Using Multisim

(b) The red waveform is the input and the blue one is the output
Problem: Draw the $V_C$ waveform

![Circuit Diagram 1](image1)

Problem:

Draw the $V_C$ waveform after closing the switch for 15ms and opening the switch.

![Circuit Diagram 2](image2)
Problem:

Draw the $V_{\text{out}}$ waveform if (a) $R=2\, \text{K}\Omega$ and $C=0.1\, \mu\text{F}$ and (b) $R=20\, \text{K}\Omega$ and $C=1\, \mu\text{F}$.

Problem:

If $V_c=4.75\, \text{V}$ the alarm will be on, how long is required when the switch is closed to $5\, \text{V}$.