Chapter 10

Capacitors and Capacitance

Capacitance

- Capacitor
  - Two conductive plates separated by insulator
  - Insulating material called dielectric
  - Conductive plates can become charged with opposite charges
Capacitance

- Capacitor can **store charge**

![Diagram of Capacitor](image)

**Definition of Capacitance**

- Amount of charge $Q$ that a capacitor can store depends on applied voltage by $Q = CV$ or $C = Q/V$ (Similar to Ohm’s Law)
- $C$ is capacitance of the capacitor and unit is the **farad (F)**
- **One farad** if it stores one coulomb of charge
- When the voltage across its terminals is one volt

**Factors affecting capacitance**
- Area $A$, spacing $d$, and dielectric $\varepsilon$
Effect of Area

- Capacitance is directly proportional to amount of charge, \( C \propto Q \)
- **Capacitance is directly proportional to plate area**, \( C \propto A \)
- Larger plate will be able to hold more charge
- If plate area is doubled, capacitance is doubled

Effect of Spacing

- Capacitance **inversely proportional to distance between plates**, \( C \propto 1/d \)
- As plates are moved closer together
  - Force of attraction between opposite charges is greater
- Double the distance between plates
  - Capacitance becomes half as much
Effect of Dielectric

- The capacitance increases if the dielectric constant or relative permittivity is large
- If a dielectric other than air is used between the plates, more charge can build up on the plates
- Permittivity represented by $\varepsilon$ (Greek letter epsilon)
  - How easy it is to establish electric flux in a material

![Diagram of capacitor with dielectric](image)

(a) $C = 200$ pF with air dielectric  
(b) $C = 1.5$ pF with high permittivity ceramic dielectric

Capacitance of a Parallel-Plate Capacitor

- Directly proportional to plate area $A$
- Inversely proportional to plate separation $d$
- Dependent on dielectric $\varepsilon$
  
  $C = \varepsilon \frac{A}{d}$

- $\varepsilon = \varepsilon_r \varepsilon_0$, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
- A farad is a very large unit
  - $\mu$F or pF

![Diagram of capacitor](image)

(a) $C = 24$ pF with air dielectric  
(b) $C = 66$ pF
Dielectric Constants

\[
\varepsilon = \varepsilon_r \varepsilon_0 \quad C = \varepsilon_r C_0
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>(\varepsilon_r) (Nominal Values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
</tr>
<tr>
<td>Air</td>
<td>1.0006</td>
</tr>
<tr>
<td>Ceramic</td>
<td>30–7500</td>
</tr>
<tr>
<td>Mica</td>
<td>5.5</td>
</tr>
<tr>
<td>Mylar</td>
<td>3</td>
</tr>
<tr>
<td>Oil</td>
<td>4</td>
</tr>
<tr>
<td>Paper (dry)</td>
<td>2.2</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Example

Compute the capacitance of a parallel-plate capacitor with plates 10cm by 20cm, separation of 5mm, and a mica dielectric.
**Electric Fields**

- Regions in which forces between charges exists
- Electric flux
  - **Electric field lines** are indicated by $\psi$ (Greek letter psi)
- Direction of this field is direction of force on a positive test charge
- Field lines never cross
- Density of lines indicate field strength

**Electric Field strength**

- **Strength of an electric field** is force that field exerts on a small test charge
  - $E = \frac{F}{Q}$
- **Electric flux density** = total flux/area
  - $D = \frac{\psi}{A}$
- Flux is due to the charge $Q$
- The number of flux lines coming from a charge is equal to the charge itself
  - $\psi = Q$
Field of a Parallel-Plate Capacitor

- To move a charge from the negative plate to the positive plate requires work
- Work = Force × distance, \( W = F \times S \)
- Voltage = Work/charge, \( E = \frac{W}{Q} \)
- \( E = \frac{V}{d} \)

Voltage Breakdown

**High voltage** causes the dielectric breakdown of a capacitor

- Force on electrons becomes very great
- Electrons are torn from orbit
- For air, breakdown occurs at a voltage gradient of 3 kV/mm
**Capacitor Voltage Rating**

Capacitors rated for maximum operating voltage (working voltage dc)

- Rating is necessary due to dielectric breakdown

**Non-ideal Effects**

**Leakage current:**

- Property of real capacitors
- Eventual discharge
- Small amount of charge “leaks” through dielectric
- Effect of leakage is modeled by a resistor
**Equivalent Series Resistance (ESR)**

- Property of real capacitors
- Sources of resistance
  - Resistance of leads
  - Contact connections between leads and plates
  - ac losses in the dielectric
- **Can be modeled as a resistance in series with the capacitor**
- ESR is so small it can be neglected for many types
- Can be a problem with electrolytic capacitors
- **Cannot be measured with an ohmmeter**
  - Need specialized test instruments

**Dielectric Absorption**

- Residual charge from remaining polarized atoms:
  - Cause residual voltage after discharge
- **Shorting resistor** may need to **be put back on to complete the discharge**
Temperature Coefficient

- Positive temperature coefficient
  - Capacitance increases with increasing temperature
- Negative temperature coefficient
  - Capacitance decreases with increasing temperature
- Zero temperature coefficient
  - Capacitance remains constant

Types of Capacitors

- Fixed capacitors, often identified by their dielectrics:
  - Ceramic, Plastic, Mica, Aluminum
  - Tantalum oxide
- Electrolytic capacitors
  - Large capacitance at low cost and have a shelf life
- Surface mount capacitors
  - Soldered directly onto printed circuit boards
  - Extremely small: High packaging density
- Variable capacitors
- Supercapacitors
Standard Capacitor Values

- Capacitors manufactured in specific standard sizes
  - 0.1 µF, 0.22 µF, 0.47 µF, etc.
  - Multiples and submultiples of above values
    - 0.1 pF, 0.22 pF, 0.47 pF, etc.

Fixed Capacitors

- **Ceramic** Capacitors
  - Permittivity varies widely
  - Values change little with temperature, voltage, or aging

- **Plastic Film** Capacitors

- **Mica** Capacitors
  - Low cost, low leakage, good stability

- **Electrolytic** Capacitors
  - Large capacitance at low cost
  - Polarized

- **Surface Mount** Capacitors
Variable Capacitors

- Used to **tune a radio**
- Stationary plates and movable plates
  - Combined and mounted on a shaft
- A trimmer or padder capacitor is used to make fine adjustments on a circuit

Supercapacitors

- Devices with enormous capacitance values
  - Values extend into the hundreds of farads and beyond
  - Also known as **ultracapacitors**
  - **Used in power sources**, GPS systems, PDAs, medical equipment, security systems
- Voltage rating typically only a few volts
Capacitors in Parallel

- Total charge on capacitors is sum of all charges
  \[ Q = CV \]
  \[ C_T E = C_1 V_1 + C_2 V_2 + C_3 V_3 \]
- **All voltages are equal** \( E = V_1 = V_2 = V_3 \)
- Total capacitance of capacitors in parallel
  - Sum of their capacitances (like resistors in series)
  \[ C_T = C_1 + C_2 + C_3 \]

Capacitors in Series

- **Same charge** appears on all capacitors \( Q = Q_1 = Q_2 = Q_3 \)
- Total \( V \): Sum of individual voltages (like resistors in parallel)
  \[ E = V = V_1 + V_2 + V_3 \]
- \[ C_T = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)} \]

\[ V = \frac{Q}{C} \]
\[ \frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \]
\[ \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]
Example: Capacitors in Series

\[ C_T = \frac{1}{1/30 \mu F + 1/60 \mu F + 1/20 \mu F} = 10 \mu F \]
Capacitor Voltage During Charging

- Voltage across a capacitor \textbf{does not change instantaneously}
- Voltage begins at zero and \textbf{gradually climbs to full voltage} (or source voltage)
- May range from nanoseconds to milliseconds
  - Depending on the resistance and capacitance

\[ R \quad \begin{cases} \text{if } v_C = 0 \\ \text{if } v_C = 0 \end{cases} \]

\[ i_C \quad E \quad v_C \]

Voltage builds as capacitor charges

\[ \text{Time} \]

Capacitor Current During Charging

- During charging
  - \textbf{Electrons move from one plate to another}
  - Current lasts only until capacitor is charged
  - Current
    - \textbf{Large initial spike to zero}

\[ R \quad \begin{cases} \text{if } i_C = 0 \\ \text{if } i_C = 0 \end{cases} \]

\[ i_C \quad E \quad v_C \]

Current decays as capacitor charges

\[ \text{Before charging} \quad \text{During charging} \quad \text{After charging} \quad \text{Time} \]
**Capacitor \( v-i \) Relationship**

\[
q = Cv_C \\
\dot{i}_C = \frac{dq}{dt} = \frac{d}{dt}(Cv_C) \\
\dot{i}_C = C\frac{dv_C}{dt} \quad (A) \\
\dot{i}_C = C\frac{\Delta v_C}{\Delta t} = C \frac{\text{rise}}{\text{run}} = C \times \text{slope of the line}
\]

**Energy Stored by a Capacitor**

- **A capacitor does not dissipate power**
- **Stored energy**: When power is transferred to a capacitor

\[
W = \int_{0}^{t} p \, dt = C\int_{0}^{t} v \frac{dv}{dt} \, dt = C\int_{0}^{V} v \, dv = \frac{1}{2}CV^2
\]

\[
\text{Energy} = \frac{1}{2}CV^2
\]
Capacitor Failures and Troubleshooting

- Reasons for capacitor’s failure
  - Excessive voltage, current, or temperature, or aging
- Test with an ohmmeter
  - Good capacitor will read low, then gradually increase to infinity
- Capacitor short
  - Meter resistance will stay low
- If capacitor is leaky
  - Reading will be lower than normal
- If capacitor is open
  - Stays at infinity

Limitation of Capacitor Testers

- Ohmmeter testing of limited usefulness
  - Does not evaluate capacitor quality issues: Such as ESR (Equivalent Series Resistance)
- Smart Tweezers™ device for testing surface mount components
- LCR/ESR component tester
Problem: Determine $C_T$

Problem: Find $V_1$ and $V_2$
Problem: Find $V_1$, $V_2$, and $V_3$