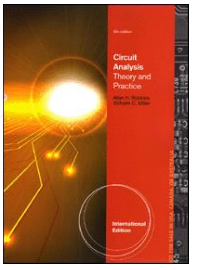



# Chapter 09

# Network Theorems



Source: Circuit Analysis: Theory and Practice ©Delmar Cengage Learning

# Outline

- Superposition Theory
- Thévenin's Theory
- Norton's Theory
- Maximum Power Transfer
- Millman's Theorem

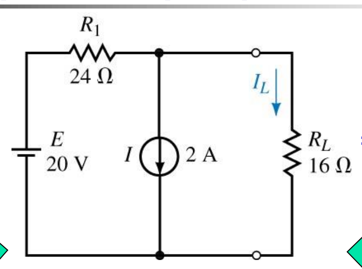
# Superposition Theorem

- **Total current through** or **voltage across** a resistor or branch
  - **Determine by adding effects due to each source acting independently**
- **Replace a voltage source with a short**
- **Replace a current source with an open**
- Find results of branches using each source independently
  - **Algebraically combine results**



# Example: Superposition Theorem

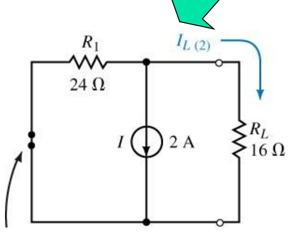
***Find $I_L$***




$I_L = I_{L(1)} + I_{L(2)} = -0.7A$

$P_{RL} = 7.84W$

$\neq P_{RL(1)} + P_{RL(2)} = 27.04W$



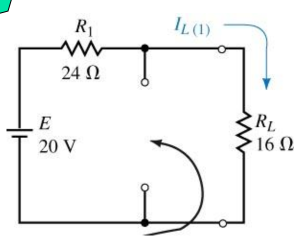

**Replace a voltage source with a short**

$I_{L(2)} = -2*24/40 = -1.2A$

$P_{(2)} = 23.04W$

$I_{L(1)} = 20/40 = 0.5A$

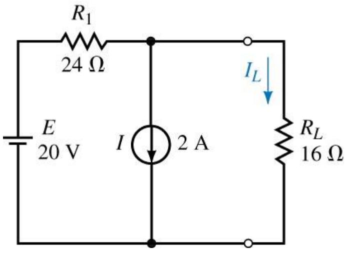
$P_{(1)} = 4W$




**Replace a current source with an open**

## Using Source Conversion

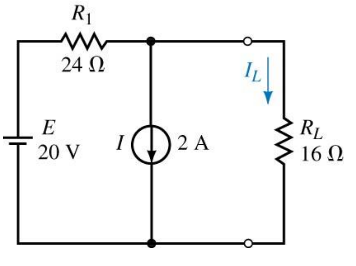
***Find $I_L$***





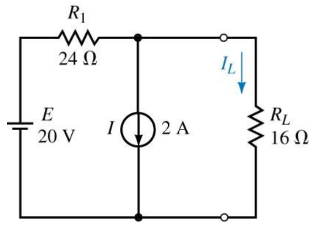

## Using Nodal Analysis

***Find $I_L$***



## Using Superposition Theorem

***Find $I_L$***



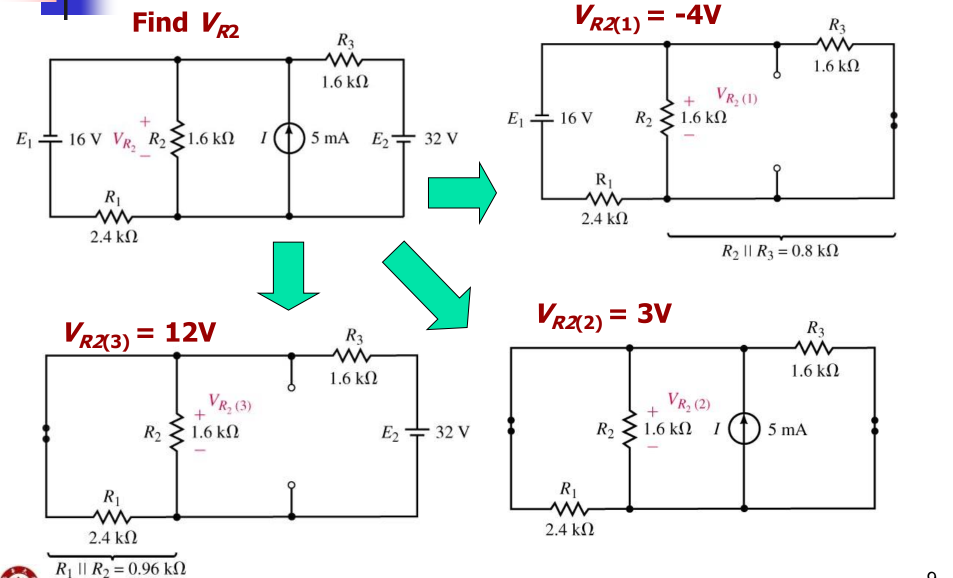



## Superposition Theorem

- **Superpositon Theorem does <u>not apply to power</u>**
  - **Power is <u>Not a linear quantity</u>**
  - Found by squaring voltage or current
- **To find power using superposition**
  - Determine voltage or current
  - Calculate power

$$P = I^2R = \frac{V^2}{R}$$

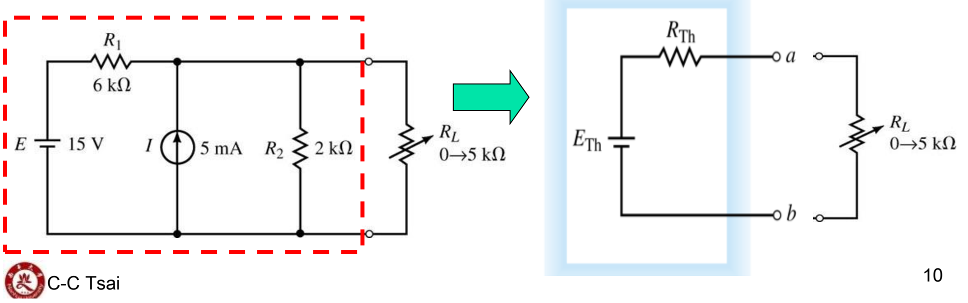
## Example: Superposition Theorem






## Thévenin's Theorem

- Any linear bilateral network can be reduced to a **simplified two-terminal circuit with a single voltage source in series with a single resistor**
- Voltage source: Thévenin equivalent voltage, $E_{Th}$.
- Series resistance: Thévenin equivalent resistance, $R_{Th}$.



## Thévenin's Theorem

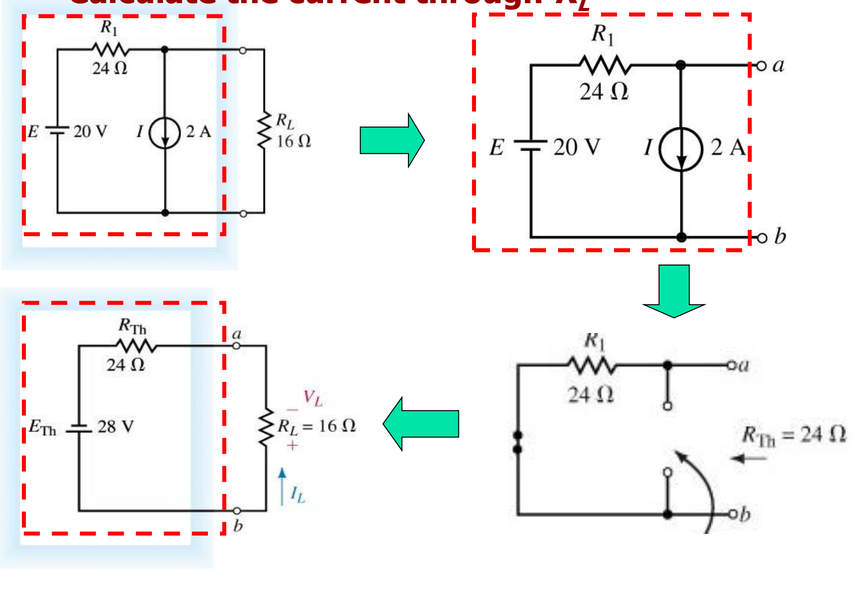
Steps to convert to a Thévenin circuit

- **Identify and remove load from circuit**
- Replace voltage sources with shorts, current sources with opens to **determine Thévenin equivalent resistance** as seen by open circuit.
- Replace sources and calculate voltage across open (If there is more than one source, Superposition theorem could be used) to **determine Thévenin equivalent voltage** as seen by open circuit.
- Draw Thévenin equivalent circuit, including load



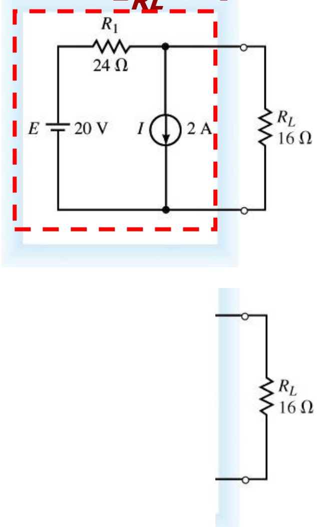
## Example1: Thévenin's Theorem

**Calculate the current through $R_L$**



# Using Thévenin's Theorem

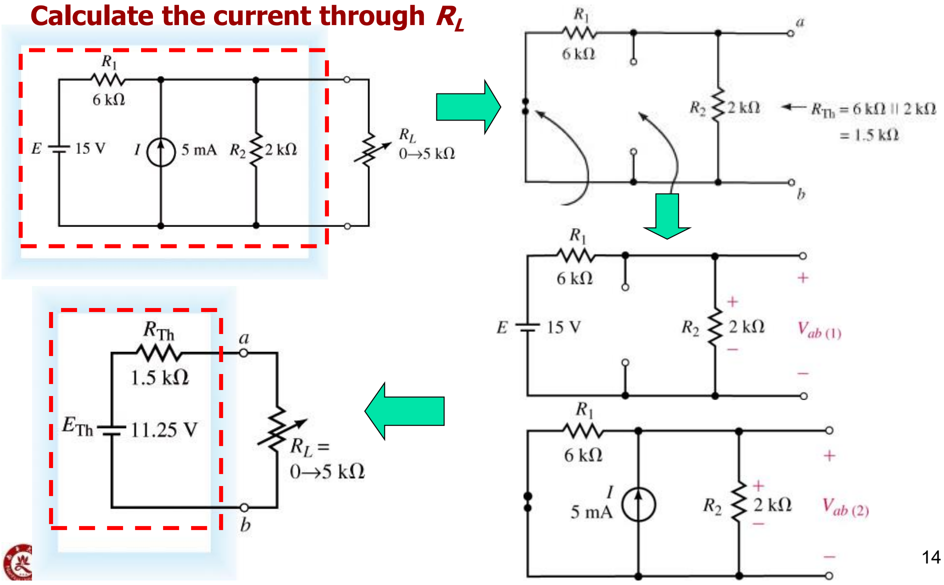
$I_{RL}$ = ?





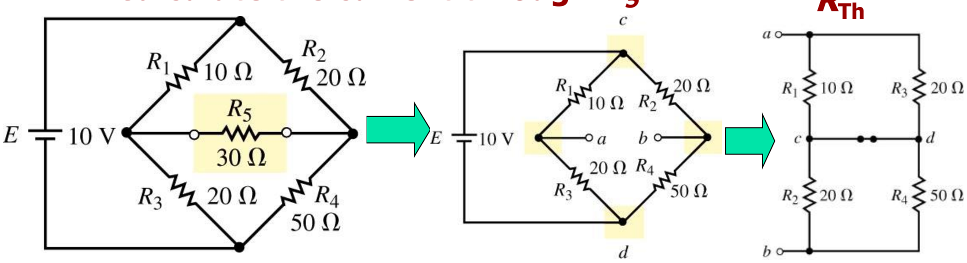

# Example2: Thévenin's Theorem

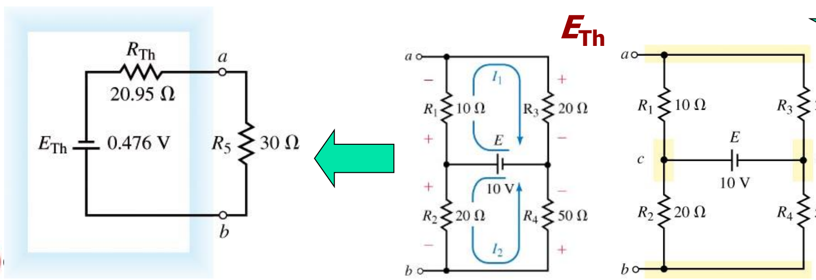
**Calculate the current through $R_L$**



# Example3: Thévenin’s Theorem

**Calculate the current through $R_5$**

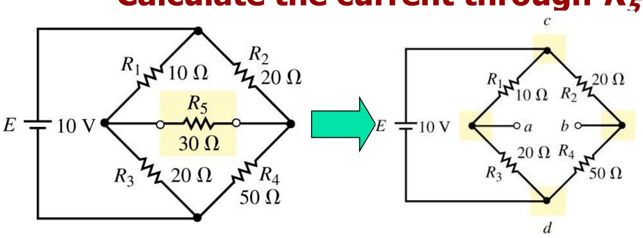


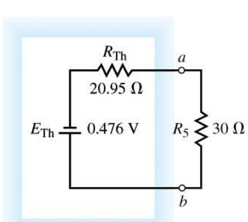






# Example3: Thévenin’s Theorem

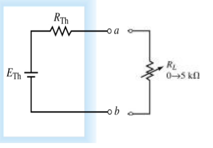
**Calculate the current through $R_5$**

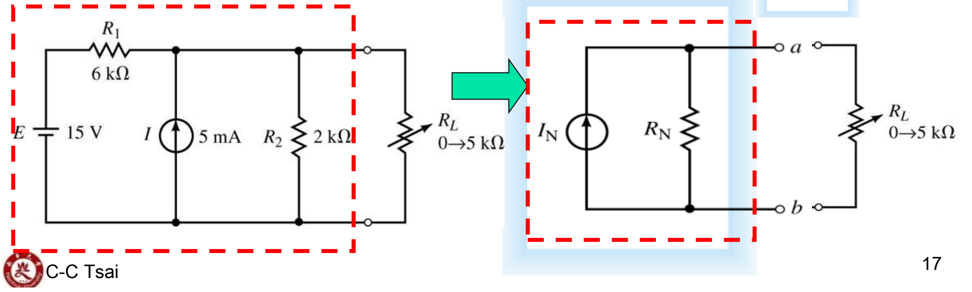


# Norton's Theorem

- Similar to Thévenin circuit
- Any linear bilateral network cab be reduced to a **two-terminal circuit with a single current source in parallel with a single resistor**
- $I_N$ is **Norton equivalent current**
- $R_N$ = $R_{Th}$ , **equivalent resistance**





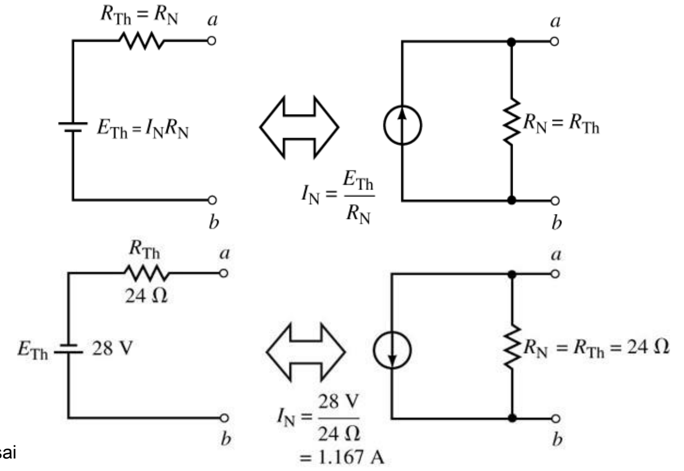

# Norton's Theorem

Steps to convert to a Norton circuit

- Identify and remove load from circuit
- Determine open-circuit resistance, i.e., **Norton equivalent resistance**.
- Replace sources and determine current that would **flow through a short place between two terminals**. This current is the **Norton equivalent current**
- For multiple sources, superposition theorem could be used
- Draw the Norton equivalent circuit including the load

# Norton's Theorem

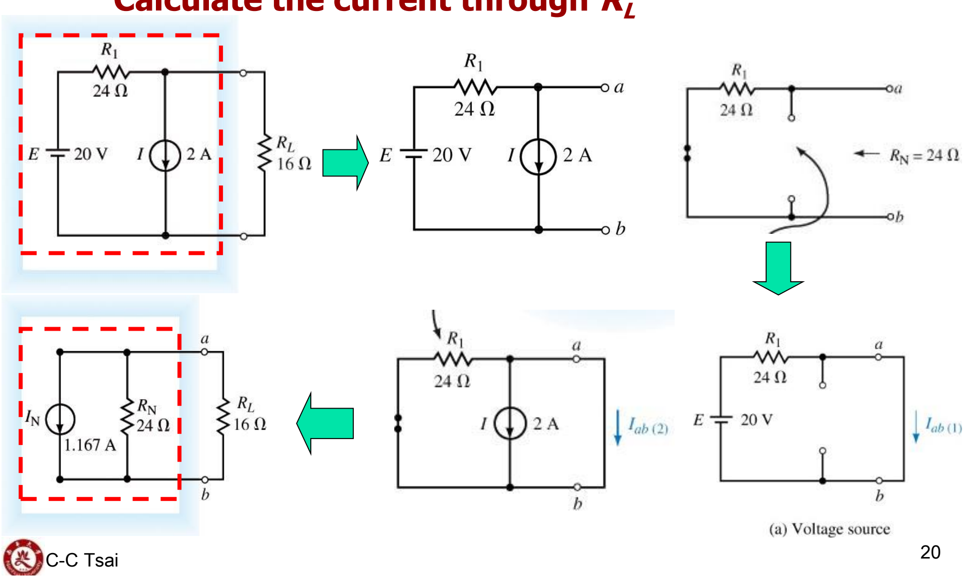
- **Norton equivalent circuit** may be determined directly from a **Thévenin circuit** by using source transformation theorem





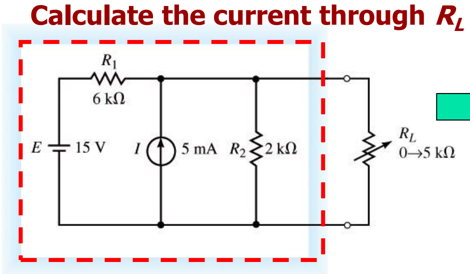

# Example1: Norton's Theorem

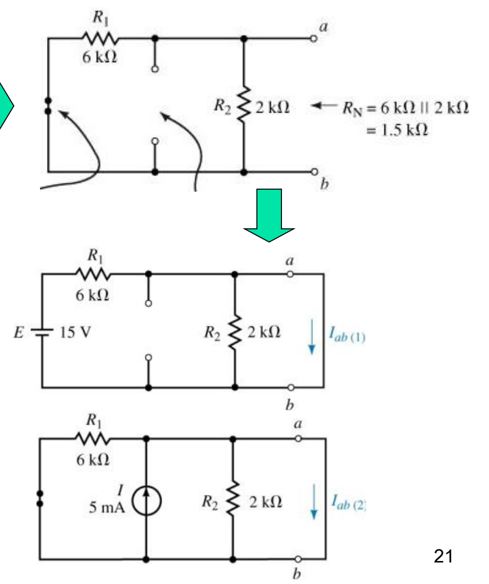
**Calculate the current through $R_L$**

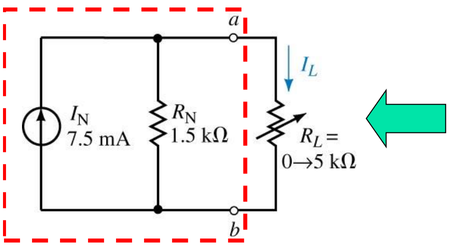


# Example2: Norton's Theorem

**Calculate the current through $R_L$**



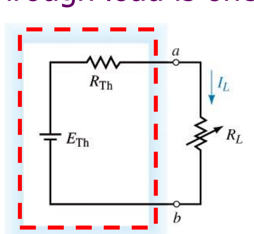



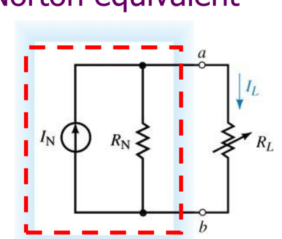





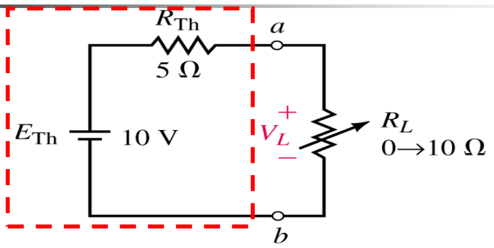

# Maximum Power Transfer

- **A load resistor will receive maximum power** from a circuit when its resistance is the same as the Thévenin (or Norton) equivalent resistance
- Calculate maximum power delivered by source to load by using $P = V^2/R$
- **Voltage across load is <u>one half of Thévenin equivalent voltage</u>**
- Current through load is one half of Norton equivalent current



## Maximum Power Transfer




$R_L$=1Ω, $I=E/(R_{th}+R_L)$=10/(5+1)=5/3, $P_{R_L}=I^2R_L$=(5/3)$^2$x1= 2.778 W

$R_L$=3Ω, $I=E/(R_{th}+R_L)$=10/(5+3)=5/4, $P_{R_L}=I^2R_L$=(5/4)$^2$x3= 4.688 W

$R_L$=4Ω, $I=E/(R_{th}+R_L)$=10/(5+4)=10/9, $P_{R_L}=I^2R_L$=(10/9)$^2$x4= 4.938 W

**$R_L$=5Ω, $I=E/(R_{th}+R_L)$=10/(5+5)=1 , $P_{R_L}=I^2R_L$=(1)$^2$ x 5 = 5 W**

$R_L$=6Ω, $I=E/(R_{th}+R_L)$=10/(5+6)=10/11, $P_{R_L}=I^2R_L$=(10/11)$^2$x6= 4.958 W

$R_L$=7Ω, $I=E/(R_{th}+R_L)$=10/(5+7)=5/6, $P_{R_L}=I^2R_L$=(5/6)$^2$x7= 4.861 W

$R_L$=9Ω, $I=E/(R_{th}+R_L)$=10/(5+9)=5/7, $P_{R_L}=I^2R_L$=(5/7)$^2$x9= 4.592 W

$R_L$=15Ω, $I=E/(R_{th}+R_L)$=10/(5+15)=1/2, $P_{R_L}=I^2R_L$=(1/2)$^2$x15= 3.75 W



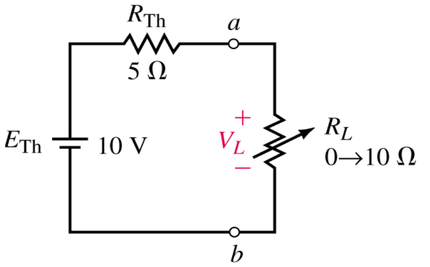
## Maximum Power Transfer

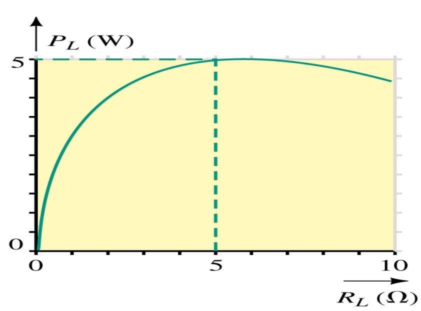
- Power across a load changes as load changes by using a variable resistance as the load

$$P_{max} = \frac{E_{Th}^{\ 2}}{4R_{Th}} = \frac{I_N^{\ 2}R_N}{4}$$

$R_L=R_{th}$ → $I=E_{th}/(2R_{th})$

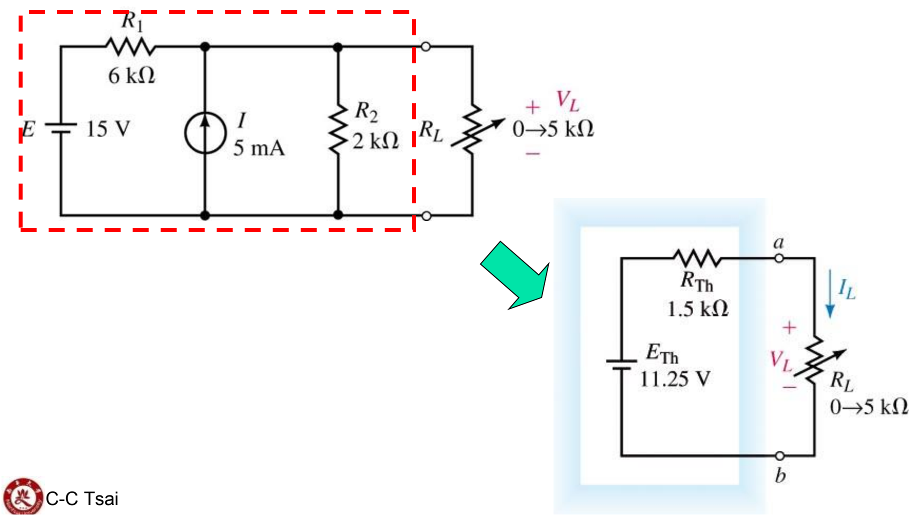
$P_{RL}=I^2R_L= E_{th}^2/(4R_{th})$



# Example: Maximum Power Transfer

**Determine the load resistance to ensure that maximum power is transferred to the load**






# Efficiency

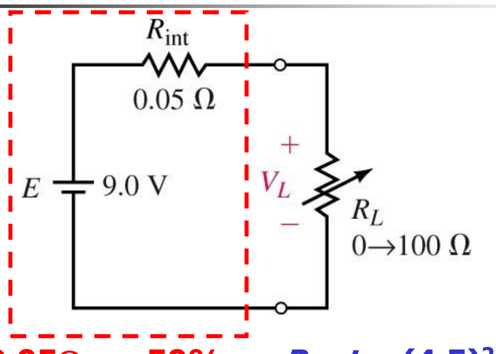
- Calculate the efficiency of maximum power transfer

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{\frac{E_{Th}^2}{4R_{Th}}}{\frac{E_{Th}^2}{2R_{Th}}} \times 100\% = 50\%$$

# Example: Efficiency




$R_L$ =0.05Ω, η=50% , *Pout* = $(4.5)^2$ / 0.05 = 405 W

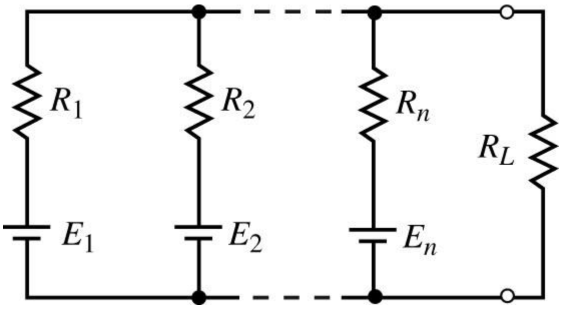
$R_L$ =50Ω, η=99.90%

$R_L$ =100Ω, η=99.95% , *Pout* = $(9)^2$ / 100 = 0.81 W



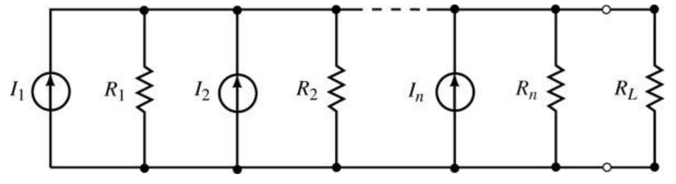
# Millman's Theorem

- Used to simplify circuits that have
  - **Several parallel-connected branches containing a voltage source and series resistance**
  - Current source and parallel resistance
  - Combination of both



## Millman's Theorem

- First, convert voltage sources into current sources
- **Equivalent current, $I_{eq}$,** is just the algebraic sum of all the parallel currents
- Next, determine **equivalent resistance, $R_{eq}$,** the parallel resistance of all the resistors
- Voltage across entire circuit may now be calculated by: $E_{eq} = I_{eq}R_{eq}$

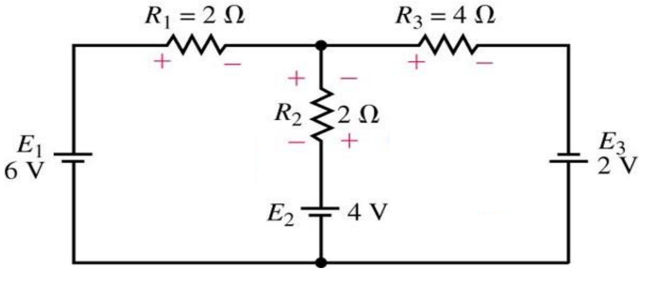





- Nodal Analysis using KVL

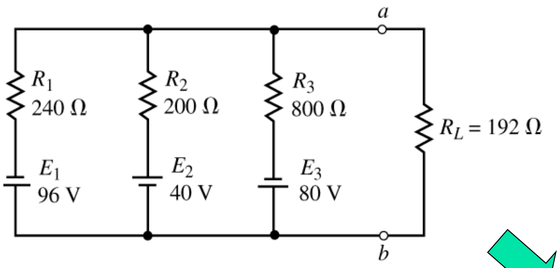
**$(V_1-6)/2 + (V_1-4)/2 + (V_1-(-2))/4= 0$**

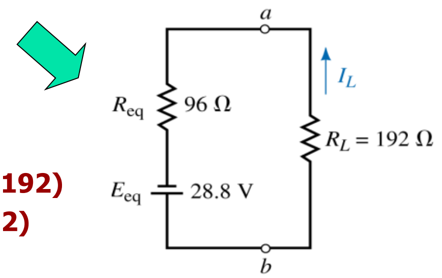
- **Using Millman's theory**



# Example: Millman's Theorem

- We can simplify a circuit as shown:



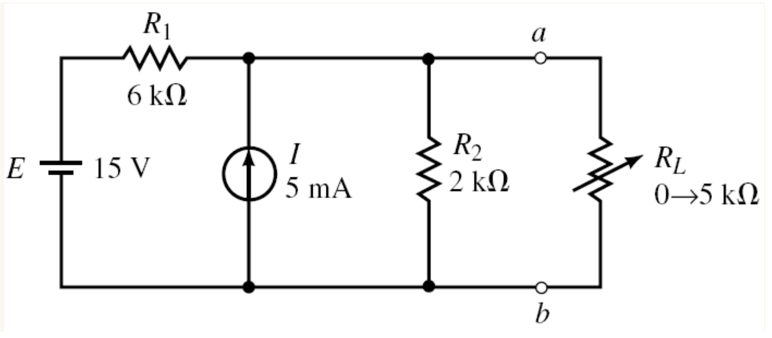




$V_{ab}$ = (-96/240+40/200-80/800+0/192) / (1/240+1/200+1/800+1/192)

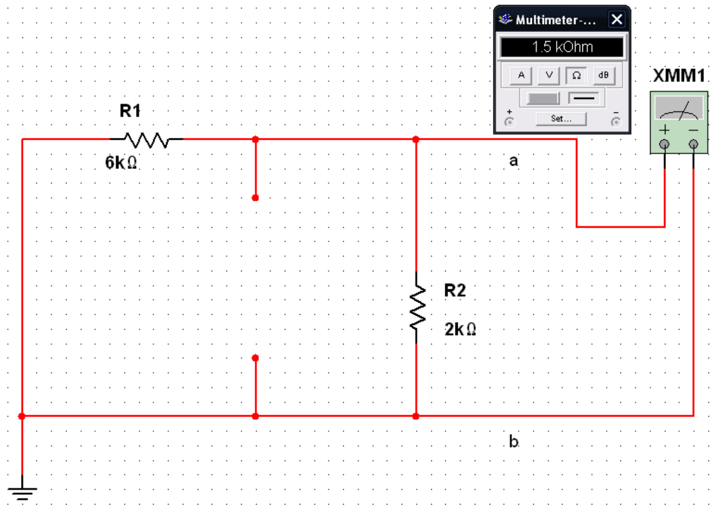


# Circuit Analysis Using Multisim

- Find both the Thévenin and the Norton equivalent circuits external to the load resistor in the circuit shown

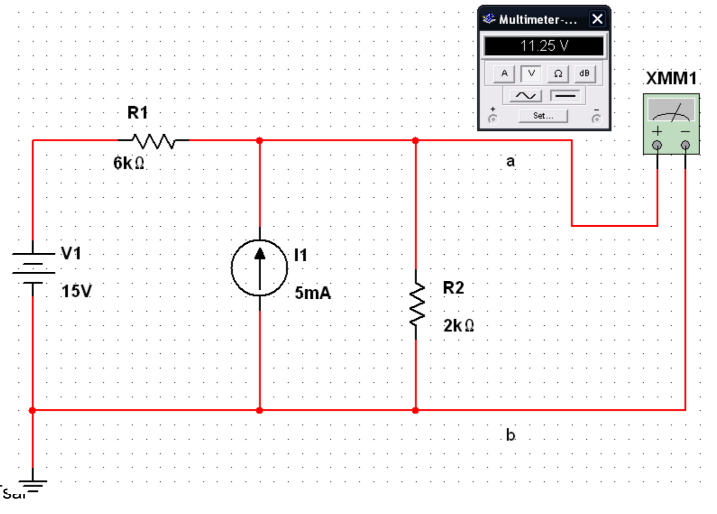


- Use a multimeter to find the equivalent resistance 1.5K Ω

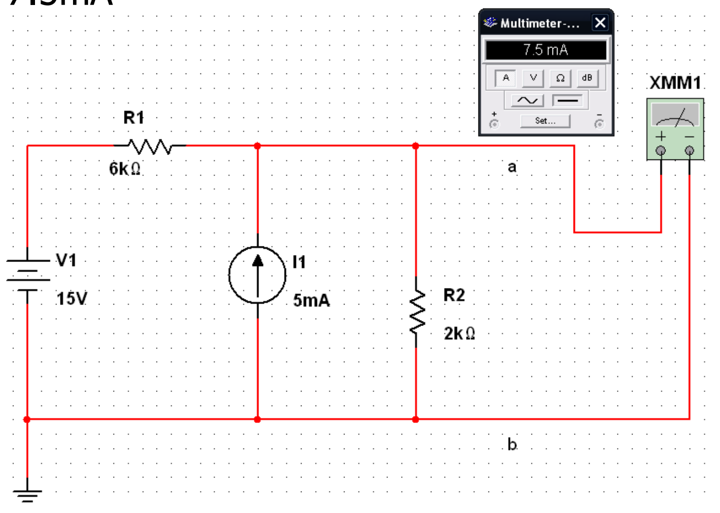





- Use a multimeter to find the open circuit voltage 11.25V

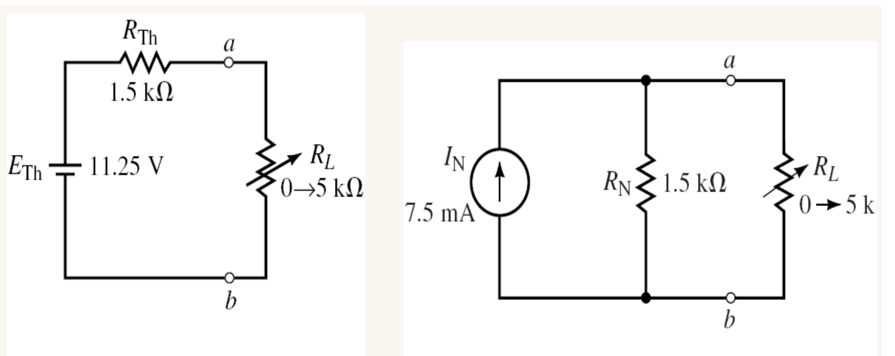


- Use a multimeter to find the short circuit current 7.5mA






- The Thévenin and the Norton equivalent circuits




(a) Thévenin equivalent circuit

(b) Norton equivalent circuit

## Kernel abilities

1. **Can use Superposition Theory for solving the unknown voltage and current of a circuit.**
2. **Can apply Thévenin's Theory for solving the unknown voltage and current of a circuit.**
3. **Can apply Norton's Theory for solving the unknown voltage and current of a circuit.**
4. **Can use Maximum Power Transfer for solving the output power.**
5. **Can recognize Millman's Theory for solving unknown voltage and current of a circuit.**